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# Measuring solar reflectance of variegated flat roofing materials using quasi-Monte Carlo method

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ABSTRACT

The Cool Roof Rating Council recommends applying the Monte Carlo (MC) method to ASTM standard C1549 to estimate the mean solar reflectance ($R$) of a variegated roofing sample. For samples with high degree of variation in the solar reflectance, the MC approach is slow in convergence. Applying proper set point of low-discrepancy sequences on the variegated roof sample can increase the convergence rate for about 40%.

We measure solar reflectance of a variegated roofing sample (gridded to $1'' \times 1''$ cells) with C1549 and calculate $R$ by averaging the measured solar reflectance. Then, we estimate $R$ using MC and quasi-Monte Carlo (QMC) technique as a function of the number of random spots on the measured sample.

To further investigate the performance of QMC, we analyze the sensitivity of the standard error of the mean reflectance of selected sample spots for a variety of simulated samples, where the range and distribution of the solar reflectance is varied. Based on the simulated and experimental results, we recommend using QMC for measuring the solar reflectance of variegated surfaces and propose an equation to estimate the required number of random spots as a function of the mix and the range of solar reflectance of samples.



## 1. Introduction

There are several instruments and standards for measuring and labeling solar reflectance and thermal emittance of roofing materials. To measure solar reflectance, pyranometer, portable solar reflectometer and spectrophotometer are widely used. ASTM E903-12: Standard Test Method for solar absorptance, reflectance, and transmittance of materials using integrating spheres [6] provides a standard for measuring near-normal beam-hemispherical spectral reflectance using a spectrophotometer equipped with an integrating sphere. Levinson et al. [17] summarized several limitations related to measurements using the spectrophotometer. For instance, the commercially available spectrophotometers can only illuminate an area of about 10 $mm^2$ of a sample. Also the sample could not be larger than 150 $cm^2$ to properly fit in instrument's port.

ASTM E1918-06: Standard Test Method for measuring solar reflectance of horizontal and low-sloped surfaces in the field [4], uses a pyranometer for measuring global solar reflectance of flat or rough horizontal and low sloped surfaces with area of at least 10 $m^2$. However, the need for clear sky and certain range of solar zenith angles (<45) limits the use of ASTM E1918.

Solar reflectance of roofing products are also measured by portable solar reflectometer. Unlike pyranometer which the reflectance of entire sample will be measured, the reflectance of surfaces is estimated by the reflectance of a considerably small area of the surface that is measured by solar reflectometer. In addition, the light source of solar reflectometer is independent from sky condition [17,18]. In comparison with spectrophotometer, this laboratory instrument can measure an area of 6.5 $cm^2$ in much shorter time.

Measurements by solar reflectometer can be conducted based on ASTM C1549-09: Standard Test Method for determination of solar reflectance near ambient temperature using a portable solar reflectometer [5]. ASTM C1549-09 is applicable for measuring the reflectance of homogeneous samples. Solar reflectance of non-uniform or variegated surfaces cannot be estimated by measuring one spot. Hence, statistical methods are required to apply for estimating solar reflectance, or solar absorptance of variegated roofing materials [23]. Therefore, the reflectance of a sufficient area should be measured to estimate the reflectance of sample ($R$) based on the reflectance of a measured area ($r$). The acceptable measurement

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area depends on the level of variegation of a sample, the method to select sample spots, and reflectance estimation accuracy.

CRRC has developed a method [3], to measure the solar reflectance of flat (or nearly-flat) variegated roofing materials using C1549-09. This method requires measuring solar reflectance of several non-overlapped random spots (Monte Carlo method) to estimate the mean solar reflectance of the sample ($R$). ANSI/CRRC S100 suggests selection of 30 random spots for this process; however, for some samples, this selected number of random points does not yield the required accuracy.

The objective of this study is to investigate the application of Quasi-Monte Carlo (QMC) method on estimating the reflectance of variegated or binary samples. QMC samplings technique is based on the quasi-random or deterministic points. In other words, the deterministic version of MC method is called QMC method.

The application of MC and QMC is tested on a sample of a fiberglass asphalt shingle. We further compare the convergence rate of MC and QMC methods for a variety of simulated samples with different levels of variegation. We provide an equation to estimate the required number of random spots as a function of the range and distribution of the solar reflectance of spots.

## 2. Theory

### 2.1. Monte Carlo methods

Monte Carlo (MC) techniques are powerful methods to high-dimensional numerical integration. Random sampling is the basis of Monte Carlo method. In MC method, random quantities are used to estimate the law of large numbers. Applying proper transformations, in many cases, make the integration domain as the s-dimensional unit cube ($I^s := [0, 1]^s$). In addition to this assumption, we presume that "integrand $f$ is square integrable over $I^s$" [20]. So according to *Monte Carlo approximation*, Eq. (1) is accepted for the integral.

$$\int_{I^s} f(u)du \approx \frac{1}{N}\sum_{n=1}^{N} f(\boldsymbol{x}_n) \tag{1}$$

where $x_1,\ldots, x_N$ represent the independent random spots selected from the uniform distribution on $I^s$. Based on the law of large numbers, we have Eq. (2). It indicates that for "almost all" set of sample points, we converge almost surely by using Monte Carlo method.

$$\int_{I^s} f(u)du = \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} f(x_n) \tag{2}$$

Acworth et al. [1] prove that the square of the error in Eq. (1) is equal to $\sigma^2(f)N^{-1}$($\sigma^2(f)$ is the variance of $f$). Hence, with overwhelming probability we have [20]:

$$\int_{I^s} f(u)du - \frac{1}{N}\sum_{n=1}^{N} f(\boldsymbol{x}_n) = 0(N^{-\frac{1}{2}}) \tag{3}$$

Eq. (3) states that the error of MC is of order $1/\sqrt{n}$. It can be concluded that we need about $10^4$ sample points to reach the error tolerance of $10^{-2}$. Also, Eq. (3) asserts that convergence rate is not a function of dimension $s$. It is the main advantage of MC method for high-dimensional problems.

On the other hand, there are several disadvantageous in using MC method. First of all, truly random selection is of crucial importance in MC method. Nevertheless generating perfect random numbers is difficult and cannot be generated through an artificial (computer) means [16]. Computer programs generate pseudo-random numbers; rather truly random numbers. However, they can satisfy standard statistical tests especially when the generated numbers are sufficiently large [16]. In this study, the random spot selected, using MATLAB *rand* function.

Second issue regarding to application of MC is related to the probabilistic error bounds for numerical integration. The last, the convergence rate is too low in many applications.

#### 2.1.1. Application of MC in measuring solar reflectance of variegated roofing materials

ASTM C1549-09 is applicable to solar reflectance measurements of uniform surfaces. To measure the solar reflectance of variegated flat surfaces that have significantly higher variation of solar reflectance, [2] applied a statistical Monte Carlo (MC) method.

The proposed technique estimates the mean solar reflectance of sample ($R$) by averaging the measured solar reflectance of several random non-overlapped spots ($\bar{r}$). Appendix A provides a summary of the MC method and resulted error of estimated mean solar reflectance. The method is used by Cool Roof Rating Council (CRRC) for measuring and labeling solar reflectance of flat or almost-flat variegated roofing materials. This method is called ANSI/CRRC S100: Standard Test Methods for determining radiative properties of materials [10].

Based on ANSI/CRRC S100, at least 30 random non-overlapped spots of a sample should be selected and measurements should be performed in accordance with ASTM C1549-09. Then the mean standard error ($\varepsilon$) of the estimated average solar reflectance, $r$, is calculated by Eq. (4).

$$\varepsilon \approx \frac{\sigma}{\sqrt{n}} \tag{4}$$

where $\sigma$ represents the standard deviation of a set of spots and $n$ is a number of random spots.

According to current CRRC requirement of measuring the solar reflectance if $\varepsilon$ is equal or less than 0.005 then $r$ can represent $R$ of sample with $\pm$0.01 accuracy (with the probability of 95%). Otherwise, the number of random spots must be increased. This procedure should be repeated until the mean standard error of spots does not exceed 0.005.

### 2.2. Quasi-Monte Carlo method

Quasi-Monte Carlo (QMC) method was introduced to address the drawbacks of MC method. MC method implements the random sample points to estimate the law of large numbers. However, QMC samplings technique is based on the quasi-random or deterministic points. In other words, the deterministic version of MC method is called QMC method. We seek deterministic points with good distribution properties that can increase the convergence rate.

A low-discrepancy point sets or sequences are correlated to provide greater uniformity [11]. An ideal form of quasi-random points are low-discrepancy sequences. Using low-discrepancy sequences can significantly increase the convergence rate compared to the convergence rate of MC (Eq. (3)). It is verified by [15] that the order of convergence is reduced to $N^{-1}(\log N)^{s-1}$ for $s = 1$ and $s = 2$. Therefore, in many cases of integrals, the QMC method will surpass the MC method. Moreover, answers produced by QMC typically are more accurate than MC approach [22].

Over the years, different low-discrepancy sequences have been derived; mostly linked to the van der Corput sequence. Various experimental studies have been performed to evaluate the performance of QMC sequences [8,25,9]. It was stated that the sample size should be sufficiently large to QMC outperforms the MC.

The Holton and Sobol [12,24] sequences are well-known low-discrepancy sequences. The outstanding features of Halton and Sobol sequences distinguish them from other quasi-random set points. The Halton sequence can produce uniform distribution for lower number of dimensions (1–10). Also, increasing $s$ (number of