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Unsteady natural convection with summer boundary conditions in a habitat at high Rayleigh number and at high time

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A B S T R A C T

A numerical study of the thermoconvective instabilities at high time in a habitat filled with Newtonian fluid is conducted. The gable roof of the habitat is subjected to a heat flux of constant density, and its side walls and floor are, respectively, adiabatic and isothermal. Based on the Boussinesq assumptions, the summer thermal and dynamic conditions are numerically studied using unsteady natural convection equations formulated with vorticity and stream-function variables. The finite volume method is used to generate the set of equations, which are solved by the iterative under-relaxation line-by-line method of Gauss–Seidel. Sudden changes in the average Nusselt number and in the extreme values of the stream functions at $Ra = 1 \times 10^8$ show that the initially unicellular flow in a pseudo-conductive regime becomes a multicellular flow with the emergence and disappearance of cells. In time and space, the change of the flow behaviour is observed more rapidly if the active walls are close to the cold horizontal wall.

1. Introduction

Most work on natural convection inside a habitat has focused on attic space, where the natural convection phenomena are a priori sensitive. Thus, a cavity with a triangular cross-section is used by many authors [1–5] to model the summer or winter thermodynamic conditions inside attic spaces. Flack [1] and Ridouane et al. [3] showed that the flow remains laminar and stable even for $Gr = 2.84 \times 10^6$ if the cavity is heated from above. However, when heated from below, the convection intensively develops as soon as $Gr = 10^5$, and the flow becomes turbulent when the Rayleigh number exceeds 1×10^9 [6]. By studying heat transfer in model saltbox and gambrel roofs, Varol et al. [7,8] observed that steady convection develops intensively at $Ra = 10^6$ in the case where the cavities are heated from below, but with summer boundary conditions, the transfers are dominated by conduction until $Ra = 4 \times 10^6$. Saha et al. [5] numerically analysed the stability of the flow when the inclined walls of the triangular cavity are subjected to periodic thermal forcing. They observed that, during the daytime heating stage, the flow is stratified for $Ra = 1.5 \times 10^6$, whereas in the night-time cooling stage, the flow becomes unstable.

These studies show that in attic spaces, under summer conditions, the contribution of transfer by conduction is dominant even for a Rayleigh number equal to 4×10^6. Because this area is adjacent to the active inclined walls where most transfers occur, it is understandable in these conditions that most dynamic fluid studies are restricted to the attic spaces. However, when the Rayleigh number may be much higher, the study must be extended to the entire habitat. In this work, the authors often used the Dirichlet thermal boundary conditions. Indeed, they subjected the upper wall to a constant or variable hot temperature that would presage the development of the system to a steady state [9] or periodically oscillating [5] state.

Therefore, we propose to study the unsteady natural convection in a pentagonal cross section cavity under a heat flux of constant density inducing more real conditions of a sunny day. The only fluid dynamics study on this type of geometry, to our knowledge, is the work of Walid and Ahmed [10] with a heat flux of constant density imposed on the floor at steady state. The present numerical study is motivated by the interest to understand the heat propagation and the air dynamics over time in a habitat with gable roof subjected to a heat from solar radiation and the vertical walls thermally insulated. It should be very useful for designers and builders who can optimise the thermal comfort with the adequate height of the walls. The moment of the total destabilisation of the system and its reorganisation with the movement of all the fluid would be a favourable time for efficient insecticide treatment because the product can easily spread throughout the area. To this end, for a roof inclination, the state of the system over time for a large Rayleigh number will be analysed, along with the effect of the height of the

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Nomenclature

$\overline{Nu}$	mean Nusselt number
$\vec{V}$	dimensionless velocity vector
A	aspect ratio, $=h/L$
C_p	specific isobaric heat capacity $\mathrm{J\,kg^{-1}\,K^{-1}}$
Gr	Grashof number, $=g\beta L^4 q/(\nu^2\lambda)$
h	side height m
ho	height of the attic space m
L	half-base m
l	third dimension m
Nu	local Nusselt number
Pr	Prandtl number, $=\nu/\alpha$
q	wall heat flux $\mathrm{W/m^2}$
Ra	Rayleigh number, $= Gr\,Pr$
S	length of an incline wall m
t	dimensionless time
T_o	temperature of the base wall K
u, v	horizontal and vertical dimensionless velocity coordinates
x, y	horizontal and vertical dimensionless coordinates

Greek Symbols

α	thermal diffusivity $\mathrm{m^2\,s^{-1}}$
β	coefficient of thermal expansion $\mathrm{K^{-1}}$
γ	angle rad
λ	thermal conductivity $\mathrm{W\,K^{-1}\,m^{-1}}$
μ	dynamic viscosity $\mathrm{kg\,m^{-1}\,s^{-1}}$
ν	kinematic viscosity $\mathrm{m^2\,s^{-1}}$
ω	dimensionless vorticity
ψ	dimensionless stream function
ψ_{min}, ψ_{max}	minimum and maximum values of ψ
ρ	density $\mathrm{kg\,m^{-3}}$
θ	dimensionless temperature, $=\lambda\,(T-T_o)/(qL)$

side wall at three aspect ratios (ratio of the height to base) on the transfer.

2. Physical model and the mathematical formulations

2.1. Physical domain

The physical system shown in Fig. 1 contains a Newtonian fluid. Its physical properties are the dynamic viscosity μ, density ρ, thermal conductivity λ and specific heat ρc_p. The inclined upper walls are subjected to a heat flux of constant density q, the side walls are adiabatic and the horizontal bottom wall is maintained at a uniform cool temperature T_o.

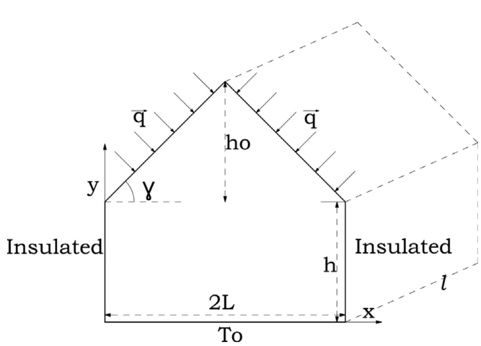

Fig. 1. Physical system.

2.2. Assumptions and equations

From time $t>0$, a constant heat flux q is applied to the inclined walls. The thermodynamic balance is broken by the appearance of buoyancy forces. To study this problem of thermoconvection, we assume that the flow is two-dimensional. We further assume that the physical properties of the fluid are constant unless its density in terms of gravity, which in a first approximation of Boussinesq hypothesis, varies linearly with temperature. The reference parameters used to make the problem dimensionless are the half-base L, $\frac{L^2}{\alpha}$ and $\frac{qL}{\lambda}$, which, respectively, represent the length, time and temperature gradient. In the stream function and vorticity formulation, the non-dimensional equations of heat, vorticity and stream function are, respectively, as follows:

$$\frac{\partial\theta}{\partial t}+\vec{\nabla}.(\vec{V}.\theta-\vec{\nabla}\theta)=0 \tag{1}$$

$$\frac{\partial\omega}{\partial t}+\vec{\nabla}.(\vec{V}.\omega-Pr.\vec{\nabla}\omega)=Pr.Ra\frac{\partial\theta}{\partial x} \tag{2}$$

$$\vec{\nabla}.(-\vec{\nabla}\psi)=\omega \tag{3}$$

where ψ and ω are such that:

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}\text{ and }\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$$

The above equations are complemented by the following initial and boundary conditions:

when $t=0$:

$$\psi(x,y,0)=\omega(x,y,0)=\theta(x,y,0)=0$$

when $t>0$:

- hydrodynamic conditions on the walls:

$$\psi=(\vec{n}.\vec{\nabla})\psi=0\text{ and }(\vec{n}.\vec{\nabla})[(\vec{n}.\vec{\nabla})\psi]=-\omega$$

- thermal conditions on the inclined walls:

$$(\vec{n}.\vec{\nabla})\theta=-1$$

- thermal conditions on the side walls:

$$(\vec{n}.\vec{\nabla})\theta=0$$

- thermal conditions on the bottom wall:

$$\theta(x,0,t)=0$$

where $\vec{n}$ is the external normal vector to each wall.

The heat energy transmitted by the active inclined walls is characterized by the Nusselt number. The local and mean Nusselt numbers of an active wall are as follows:

$$N_u=\frac{1}{\theta_p}$$

$$\overline{N_u}=\frac{1}{S}\int_S N_u ds$$

where θ_p is the instantaneous local temperature of the wall.