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A new analytical model for short-time response of vertical ground heat exchangers using equivalent diameter method

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ABSTRACT

The composite medium infinite line source model (CMILS) is an attractive model to describe the short time thermal processes of borehole ground heat exchanges. However, this model is very complicated and has a lower computation efficiency which limits its practical applications. In this paper, an equivalent diameter method to the CMILS model is introduced and a new analytical model is developed. The short-term performance of the new model was verified by comparison with three noteworthy analytical solutions and reported experimental data. The impact of the shank spacing of U-tube legs was studied through the new analytical model explicitly. Simulation studies of the equivalent radius were also performed to get the best approach of calculating the equivalent radius with a higher accuracy.

1. Introduction

Modeling short time response of vertical ground heat exchanger (GHE) has become the major focus of GHE related research in recent years [1–3]. The short-time response has significant influence on the performance of ground source heat pump systems. It is critical in (1) the heat flux build-up stages; (2) calculating hourly or sub-hourly thermal energy use; and (3) reducing the thermal response test (TRT) duration [4]. Existing models of GHE for short-time response were based on analytical, numerical method, or a combination of both. However, most numerical approaches, such as finite-difference and finite-element methods, have the disadvantages of time-consuming for year-round and life-cycle simulations of GHEs, and lack of flexibility for different geometry configuration, which limit its engineering applications. Due to these reasons, analytical solutions to calculate the short-time response of GHE have generated a lot of interest from researchers [1,5–13].

Gu and O'Neal [14] developed an analytical short-time response solution by assuming the GHE as a cylindrical source in an infinite composite region. In order to solve the transient borehole heat transfer problem, the generalized orthogonal expansion technique was adopted which required calculating the multiple eigenvalues. Beier and Smith [15] obtained a time domain solution by a numerical inversion through solving the borehole transfer problem in the Laplace domain. Young [16] drew an analogy between buried electric cable and vertical borehole heat exchanger, and developed an analytical Borehole Fluid Thermal Mass (BFTM) solution by modifying the classical Buried Electric Cable (BEC) solution. The thermal capacity of the grout to the equivalent-diameter pipe was lumped to a thermal capacity in this solution. For this problem Young used a grout allocation factor to deal with. Lamarche and Beauchamp [6] used an equivalent diameter approach and developed an exact analytical solution. Taking into account the different thermal properties of grout and soil, the heat transfer problem was solved by assuming a steady heat-flux condition across the equivalent cylinder boundary. However, this solution ignores the thermal capacity of the fluid in the U-tube. Javed and Claesson [8,9,17] developed an analytical solution which considered the thermal capacities, the thermal resistances and the thermal properties of all the borehole elements. This model was easily incorporated in building energy simulation software to optimize the operation and control of GSHP systems. Li [10,11,18] developed a new composite media infinite line source (CMILS) model based on Jaeger's infinite instantaneous line source solution in composite cylindrical media. This model approximated the two legs of U-tube as two infinite line source located in a composite cylindrical media of grout and soil. The performance of this model, which was appropriate for the short time response, was validated by comparison of simulation results and experimental data. However, the CMILS model is fairly sophisticated, and its computation is time-consuming even with advanced numerical integration methods.

In this paper, a new analytical model is proposed by introducing the equivalent diameter method to CMILS model for simplifying its form and improving its computation speed. The new analytical

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Nomenclature

a_i	Thermal diffusivity $i = 1, 2, s, g \cdot \left(\mathrm{m^2/s}\right)$
a	Dimensionless thermal diffusivity $\left(= \sqrt{a_1/a_2}\right)$
c	Specific heat capacity (J/(kg K))
C	Volumetric heat capacity (J/(m^3 K))
C_p	Thermal capacity per unit length [J/(m K)] ($C_p = \pi {r_e}^2 \rho c$)
r_0	Half distance between centers of U-tube pipes (m)
f, g, φ, ψ	Combined function used in CMILS model
Fo	Fourier number
h_p	Convective heat transfer coefficient (W/(m^2 K))
H	Height of borehole (m)
J_n	Bessel function of the first kind of order n
k_i	Thermal conductivity $i = 1, 2, p, s, g \cdot \left(\mathrm{W/(mK)}\right)$
k	Dimensionless thermal conductivity $\left(= k_2/k_1\right)$
q_l	Heat flux per unit length $\left(\mathrm{W/m}\right)$
$\left(r', \theta'\right)$	Cylindrical coordinate of the infinite line source
r	Radius (m)
r_b	Borehole radius (m)
r_e	Equivalent radius (m)
r_{po}	Outer radius of U-tube (m)
r_{pi}	Inner radius of U-tube (m)
R, R'	Ratio of radius
R_g	Thermal resistance of grout ((m K)/W)
R_p	Thermal resistance of U-pipes ((m K)/W)
T	Time (s)
T_0	Initial temperature (K)
$\bar{T}$	Average temperature of time t at radius r (K)
T_i	Temperature of time t at point $\left(r, \theta\right)$ of region i (K)
Y_n	Bessel function of the second kind of order n
P	Density (kg/m^3)
θ	Angle (rad)

Subscripts

1	Regions of $r \le r_b$
2	Regions of $r > r_b$
P	U-pipe
G	Grout
S	Soil or ground

model can be easily implemented by computer program, and its calculating speed is faster than CMILS model. The effectiveness of this model is evaluated by comparison with three noteworthy analytical models using simulated and experimental data. Furthermore, the impacts of equivalent radius and the shank spacing of U-tube legs are discussed to study the performance of the new analytical solution.

2. Developing the CMILS model

2.1. Composite media infinite line source model

Fig. 1 shows there is an infinite line source $\left(r', \theta'\right)$ in cylindrical coordinate which is parallel to the height (vertical) direction, z-axis. The region $r \le r_b$ is of one medium having conductivity k_1 and diffusivity a_1, and region $r > r_b$ is another, the corresponding quantities are k_2 and a_2. The initial temperature was assumed uniform and to be zero for all mediums, i.e. fluid, grout, and soil. The infinite line source in composite media will continuously release a constant heat flux q_l into the solid from the initial time, and the temperature response at point $\left(r, \theta\right)$ can be obtained from the following expressions [10,18]:

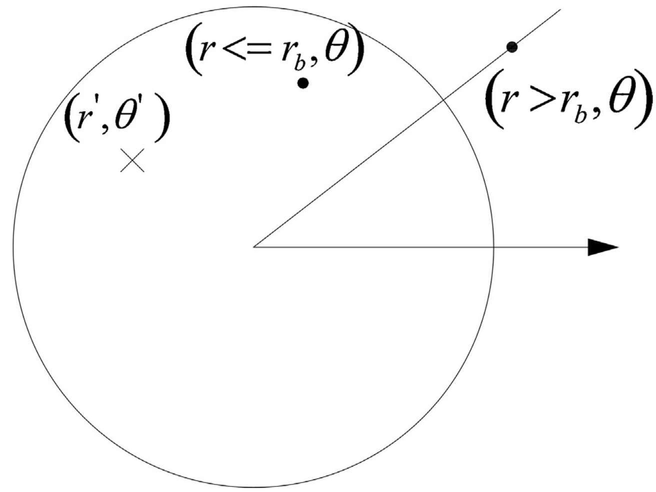

Fig. 1. Schematic layout of the CMILS model.

$$\begin{cases} T_1\left(t, r, \theta\right) = \dfrac{q_l}{2\pi k_1} \displaystyle\sum_{n=-\infty}^{+\infty} \cos n(\theta - \theta') \int_0^{+\infty} \left(1 - e^{-v^2 \mathrm{Fo}}\right) \dfrac{J_n(vR) J_n\left(vR'\right)\left(\phi g - \psi f\right)}{v\left(\phi^2 + \psi^2\right)} dv & , r \le r_b \\ T_2\left(t, r, \theta\right) = \dfrac{q_l}{\pi^2} \displaystyle\sum_{n=-\infty}^{+\infty} \cos n(\theta - \theta') \int_0^{+\infty} \left(1 - e^{-v^2 \mathrm{Fo}}\right) \dfrac{J_n\left(vR'\right)\left[\psi J_n(avR) - \phi Y_n(avR)\right]}{v^2\left(\phi^2 + \psi^2\right)} dv & , r > r_b \end{cases} \quad (1)$$

where

$$\mathrm{Fo} = \frac{a_1 t}{r_b^2}, R = \frac{r}{r_b}, R' = \frac{r'}{r_b}$$

and

$$\phi = akJ_n(v)J'_n(av) - J'_n(v)J_n(av)$$

$$\psi = akJ_n(v)Y'_n(av) - J'_n(v)Y_n(av)$$

$$f = akY_n(v)J'_n(av) - Y'_n(v)J_n(av)$$

$$g = akY_n(v)Y'_n(av) - Y'_n(v)Y_n(av)$$

Subscripts 1 and 2 denote regions of $r \le r_b$ and $r > r_b$; J_n and Y_n denote the Bessel functions of first kind and second kind of order n; k_i and a_i ($i = 1, 2$) are thermal conductivity and thermal diffusivity respectively; a and k are dimensionless variables $a = \sqrt{a_1/a_2}$, $k = k_2/k_1$.

2.2. The new analytical model using equivalent diameter method

In most practical cases, the heat transfer of GHE is assumed to be radial only. A simplified method to modeling the GHE is to replace the adjacent legs of U-tube with a single equivalent diameter pipe (Fig. 2). The heat transfer from the U-tube is then approximated by the heat transfer from the hypothetical diameter pipe through which the heat carrying fluid circulates. Thereby the equivalent diameter method [6,9,12,14,15,17,19–21] simplified the complicated multi-dimensional problem to a one dimensional model. Various methods [22–25] have been suggested to approximate the equivalent radius of the pipe.