Energy and Buildings 98 (2015) 66–73

Contents lists available at ScienceDirect

Energy and Buildings

journal homepage: www.elsevier.com/locate/enbuild

Theoretical and experimental study of heat transfer in wall heating panels

Ružica Todorović*, Miloš Banjac, Milan Gojak

Department of Thermomechanics, Faculty of Mechanical Engineering, University of Belgrade, Belgrade 11120, Serbia

A R T I C L E I N F O

Article history:
Available online 1 November 2014

Keywords:
Wall heating panel
Panel surface temperature
Pipe diameter
Spacing between pipes
Volumetric flow, Inlet water temperature

A B S T R A C T

This paper presents the analysis of the impact of basic geometric parameters – pipe diameter and the spacing between the pipes, and technological parameters – volumetric flow and heating water inlet temperature, on a temperature field on the wall heating panel surfaces, formed during the steady conditions. The analysis was conducted by using the Mikheev's criterion equation, which describes the process of convection heat transfer from the heating fluid to the inner surface of the pipe, and Faxen–Rydberg–Huber expression, which describes the two-dimensional temperature field in the wall, with series of embedded heated pipes having uniform temperature. The quality of obtained theoretical results was checked experimentally, by measurements conducted using thermal imaging camera. Based on the analysis of the obtained results, and depending on the impact they have on the increase of the panel surface average temperature, the ranking of analyzed parameters was performed.

1. Introduction

Excessive and ever increasing use of fossil fuels, approaching the end of their reserves, and the consequences that are manifested in the form of global warming of the Earth, demanded urgent measures of rational attitude toward energy, and increasing use of renewable energy sources. In accordance with this process, intensive use of heat pumps started, as the most affordable system that enables the efficient use of RES. The use of these systems, as well as other low-temperature and therefore energy-efficient systems, re-focused the attention on the panel heating systems, as the most energy efficient, low-temperature subsystems for heat transfer into heated space. High energy efficiency of both subsystems – heat pump and wall panels, along with the simultaneous use of RES, led to the situation that these heating and cooling systems are today installed in over 50% of new residential buildings in Europe [1]. In Korea, this system is applied even in 90% of facilities [2], and its use has been significantly increased in China and in other countries [3].

Precisely because of this widespread use of panel heating systems, this paper presents the performed analysis of the impact of basic geometric parameters – pipe diameter and the spacing between the pipes, and technological parameters – volumetric flow and heating water inlet temperature, on the temperature field formed in the wall heating panel, during the steady-state conditions.

The analysis was conducted using the analytical expression, which can determine the temperature field in the wall with series of embedded heated pipes having uniform temperature, known as Faxen–Rydberg–Huber expression. The experimental verification of the accuracy of obtained theoretical results was conducted by measuring the established temperature field, i.e. average temperature at the front wall panel surface on the experimental installation.

2. Faxen–Rydberg–Huber expression

Heat conduction in the panel, in which the uniformly heated pipes are evenly spaced, can be described by Fourier's partial differential equation for steady two-dimensional heat conduction through an infinite plane wall, with the heat line sources. In the literature, Faxen–Rydberg–Huber expression is indicated as the least complicated solution to this problem.

Faxen–Rydberg–Huber expression for determining the steady temperature field in the panel cross section is derived under the assumption that the individual panel layers are made of the material having homogeneous and isotropic structure, that the distance between the pipes is much bigger than its diameter, that the temperature of the outer pipe surface is constant and that the air

---

* Corresponding author at: University of Belgrade, Department of Thermomechanics, Faculty of Mechanical Engineering, Kraljice Marije 16, 11120 Belgrade, Serbia. Tel.: +381 11 3302241; fax: +381 11 3370364.

E-mail address: rtodorovic@mas.bg.ac.rs (R. Todorović).

http://dx.doi.org/10.1016/j.enbuild.2014.10.065
0378-7788/

Nomenclature

A	variable in Faxen–Rydberg–Huber expression (°C)
a	width (m)
Bi	Biot number
b	height (m)
d	pipe diameter (m)
g_1, g_2	variables in Faxen–Rydberg–Huber expression
G_1, G_2	variables in Faxen–Rydberg–Huber expression
h_t	average total heat transfer coefficient (W/(m^2 K))
k, k'	ostensible overall heat transfer coefficients (W/(m^2 K))
K_0	correction factor in the transition regime flow of fluid through the pipe
j	member of the series
l	spacing between pipes (m)
l_p	length of the panel pipe in z direction (m)
Nu	Nusselt number
Pr	Prandtl number
r	pipe radius (m)
q_v	volumetric flow (m^3/s)
x, y, z	position co-ordinates (m)
Greek symbols	
δ	thickness (m)
ε	surface emissivity
θ	temperature difference (°C)
λ	thermal conductivity (W/(mK))
ϑ	temperature (°C)
Subscripts	
a	Air
bw	brick wall
hp	thermal plaster
in	pipe inlet
m	average conditions
o	surrounding surfaces
pi	inner pipe surface
pl	lime-cement plaster
po	outer pipe surface
sp	panel surface
w	water
1	heated room
2	adjacent room

temperatures in the rooms on both sides of the panel are equal. According to [4–6], this expression has the following form:

$$\theta = \frac{A\cdot\pi}{l}(-G_1 y - |y| - G_2) + A\sum_{j=1}^{\infty}\frac{1}{j}\left[e^{-2\pi j\frac{|y|}{l}} + g_1(j)e^{-2\pi j\frac{y}{l}} + g_2(j)e^{-2\pi j\frac{y}{l}}\right]\cos\left(2\pi j\frac{x}{l}\right) \quad (1)$$

In the expression (1) G_1 and G_2 mark the relations:

$$G_1 = \frac{k_1 - k_2}{k_1 + k_2} \text{ and } G_2 = -\frac{2\lambda_{hp}}{k_1 - k_2} \quad (2)$$

while $\theta = \vartheta - \vartheta_{a,1}$ is the difference between the panel temperature $\vartheta = \vartheta(x, y)$, at the point defined by coordinates (x, y) and the air temperature in the room $\vartheta_{a,1}$.

Also, in the expressions (2), ostensible overall heat transfer coefficients from the panel plane on which the vertical pipe axes lie – panel “core” – to the air in the heated and adjacent rooms, are defined as:

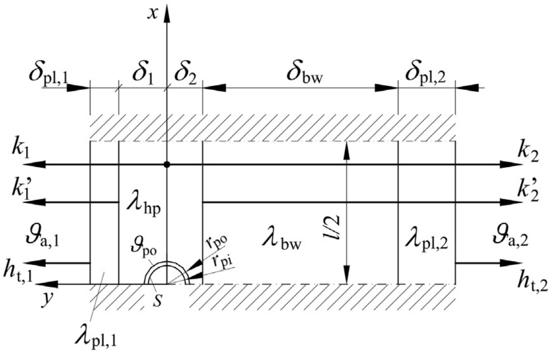

Fig. 1. Single wall heating panel element, with the ostensible overall heat transfer coefficients.

$$k_1 = \frac{1}{(\delta_1/\lambda_{hp}) + (\delta_{pl,1}/\lambda_{pl,1}) + (1/h_{t,1})} \quad (3)$$

$$k_2 = \frac{1}{(\delta_2/\lambda_{hp}) + (\delta_{bw}/\lambda_{bw}) + (\delta_{pl,2}/\lambda_{pl,2}) + (1/h_{t,2})}, \quad (4)$$

i.e. the ostensible coefficients of overall heat transfer from both surfaces of the panel layer in which heating pipes are placed (Fig. 1) are defined as:

$$k_1' = \frac{1}{(\delta_{pl,1}/\lambda_{pl,1}) + (1/h_{t,1})}, \quad (5)$$

$$k_2' = \frac{1}{(\delta_{bw}/\lambda_{bw}) + (\delta_{pl,2}/\lambda_{pl,2}) + (1/h_{t,2})}, \quad (6)$$

Introduced coefficients exist in the system of two equations on the basis of which variables $g_1(j)$ and $g_2(j)$ are determined, and that within the dimensionless parameters – Biot numbers: $Bi_1 = k_1' \cdot \delta_1/\lambda_{hp}$ and $Bi_2 = k_2' \cdot \delta_2/\lambda_{hp}$:

$$\left(Bi_1 - 2\pi j\frac{\delta_1}{l}\right)[1 + g_1(j)]e^{-4\pi j\frac{\delta_1}{l}} + \left(Bi_1 + 2\pi j\frac{\delta_1}{l}\right)g_2(j) = 0 \quad (7)$$

$$\left(Bi_2 - 2\pi j\frac{\delta_2}{l}\right)[1 + g_2(j)]e^{-4\pi j\frac{\delta_2}{l}} + \left(Bi_2 + 2\pi j\frac{\delta_2}{l}\right)g_1(j) = 0 \quad (8)$$

The variable A is determined by the dependency:

$$\frac{\theta_{po,m}}{A} = \ln\frac{l}{d_{po}\pi} - G_2\frac{\pi}{l} + \sum_{j=1}^{\infty}\frac{g_1(j) + g_2(j)}{j} \quad (9)$$

Average temperature at the front panel surface is determined based on the equality of heat flow that, in steady conditions, passes from the panel “core” to the air in the heated room and heat flow that crosses from the front panel surface to the air in the same room:

$$\theta_{sp1,m} = \theta_{y=0,m}\frac{k_1}{h_{t,1}} \quad (10)$$

3. Parametric analysis of the temperature field

Analysis of geometrical and technological parameter impact on the temperature field was carried out using Faxen–Rydberg–Huber expression (1), for four series of theoretical calculations. In each of the four calculation series the impact of one parameter on the temperature field was established.

Geometrical parameters the impact of which was analyzed were as follows: