Energy and Buildings 79 (2014) 191–201

Contents lists available at ScienceDirect

Energy and Buildings

journal homepage: www.elsevier.com/locate/enbuild

An improved lumped parameter method for building thermal modelling

C.P. Underwood*

Faculty of Engineering and Environment, University of Northumbria, UK

ARTICLE INFO

Article history:
Received 17 July 2013
Received in revised form 30 April 2014
Accepted 2 May 2014
Available online 10 May 2014

Keywords:
Dynamic thermal modelling
Building response
Lumped parameter modelling
Optimisation

ABSTRACT

In this work an improved method for the simplified modelling of the thermal response of building elements has been developed based on a 5-parameter second-order lumped parameter model. Previous methods generate the parameters of these models either analytically or by using single objective function optimisation with respect to a reference model. The analytical methods can be complex and inflexible and the single objective function method lacks generality. In this work, a multiple objective function optimisation method is used with a reference model. Error functions are defined at both internal and external surfaces of the construction element whose model is to be fitted and the resistance and capacitance distributions are adjusted until the error functions reach a minimum. Parametric results for a wide range (45) of construction element types have been presented. Tests have been carried out using a range of both random and periodic excitations in weather and internal heat flux variables resulting in a comparison between the simplified model and the reference model. Results show that the simplified model provides an excellent approximation to the reference model whilst also providing a reduction in computational cost of at least 30%.

1. Introduction

In 2002 Gouda et al. [1] developed a simplified method for the dynamic thermal modelling of single-layer and multi-layer construction elements. They used an optimisation algorithm to find the five required parameters of the simplified model by matching its dynamic response to a high-order reference model. The work was limited in three respects:

- A unit step response was used as the excitation variable for the simplified model parameter fitting whereas excitations in practice vary continuously.
- The results were based on excitations applied individually to both heat flux and temperature at one surface only using a single objective function search algorithm whereas in practice, both internal and external surfaces would be subject to simultaneous excitations of more than one variable.
- Only two sets of results were published making it difficult for other users to make use of the simplified model.

In this work an improved method is proposed for the extraction of the simplified model parameters based on a multiple objective function search algorithm (i.e. objective functions simultaneously applied to both inside and outside surfaces) and the use of a reference model consisting of a rigorous finite-difference method. Extensive sets of results are generated for a range of common construction elements and a sample of these elements are tested in the context of a simple room enclosure model which alternately uses the simplified model and the more rigorous reference model for its construction elements.

2. Review

The application of lumped parameter modelling methods to building dynamic thermal response is motivated by the desire to find simpler and, hence, computationally less 'expensive' methods for the analysis building thermal energy response. Approaches broadly fall into two categories:

- Lumped parameter construction element models from which whole room models may be constructed [1–3]
- Lumped parameter whole room models [4–8]

* Tel.: +44 191 227 3533.
E-mail address: chris.underwood@northumbria.ac.uk

http://dx.doi.org/10.1016/j.enbuild.2014.05.001
0378-7788/

List of symbols

A	area (m^2)
C	thermal capacity per unit area ($J\,m^{-2}\,K^{-1}$)
c	specific heat capacity ($J\,kg^{-1}\,K^{-1}$)
F	Fourier number
f	thermal resistance rationing factor
g	thermal capacity rationing factor
h	surface convection coefficient ($W\,m^{-2}\,K^{-1}$)
k	thermal conductivity ($W\,m^{-1}\,K^{-1}$)
L	number of layers of material
m	mass flow rate ($kg\,s^{-1}$)
Q	heat transfer (W)
R	thermal resistance ($m^2\,K\,W^{-1}$)
T	temperature (°C, K)
T'	sol-air temperature (external), rad-air temperature (internal) (°C)
t	time (s)
W	weighting factor
x	distance (m)

Greek

α	thermal diffusivity ($m^2\,s^{-1} = k/\rho\cdot c$)
Δt	time step increment (s)
Δx	spatial increment (m)
ε	root-mean-square error
ρ	density ($kg\,m^{-3}$)
$\sum C$	total element thermal capacity per unit area ($J\,m^{-2}\,K^{-1}$)
$\sum R$	total element thermal resistance ($m^2\,K\,W^{-1}$)

Subscripts and superscripts

a	air, material ref. 'a'
b	material ref. 'b'
c	convection
i	layer node index
i	internal (space)
m	middle position
n	time row index
o	outside, exterior
r	radiant, solar radiation
s	surface
s	surface index number
upper	upper bound limit

Though the differences between the two approaches are rather subtle (since models of individual construction elements are almost always used as a basis for grouping or aggregating into whole room models), the treatment of individual elements usually provide greater detail in modelling information such as individual surface temperatures which can be important when dealing with radiant sources, etc.

Lorenz and Masy [2] were among the first to propose a simplified lumped parameter approach to building response modelling using a first-order model consisting of two resistances and one capacitor. Gouda et al. [1] demonstrated improved accuracy using a second-order model in which each construction element is described using three resistances and two capacitances. These approaches to modelling were often referred to as 'analogue circuit' models due to their connotation with electric circuits (i.e. see Fig. 1 in Section 4). Fraisse et al. [3] also compared first- and second-order element models (the latter referred to as a '3r2c' model) and went further to propose a fourth-order '3r4c' model with aggregated resistances. Like Lorenz and Masy [2], they propose an analytical method for deriving the parameters of the model (essentially, the distribution of resistance and capacitance values throughout the 'circuit') whereas Gouda et al. [1] used an optimisation method to determine the parameters with reference to a rigorous reference model.

Crabb et al. [4], Tindale [5] and others [6–8] have applied the lumped parameter approach to the formulation of low-order whole room models by casting the capacitance parameter over the higher capacity elements of a room (external walls, solid floors, etc.) and using algebraic heat balances for the lower capacity room elements (demountable partitions, etc.). Tindale [5] attempted this using a second-order room model but found that it provided unacceptable results for rooms with very high thermal capacity (i.e. 'traditional' construction). He corrected this by introducing a third 'equivalent' room capacitance which required an inconvenient method for its parameterisation.

Though low-order whole room models offer very low computational demands and simplicity, there remain questions over the accuracy of these models particularly over long time horizons and they tend to provide less modelling information (i.e. individual and accurate element surface temperatures) essential in many lines of design enquiry. For this reason, it is argued that room models constructed from second-order (or higher) construction element descriptions provide greater accuracy and detail whilst retaining some of the key advantages of simplicity and low computational demand and are, therefore, to be preferred other than for approximate and early feasibility simulation studies.

The key advantages of lumped parameter building modelling are those of simplicity, transparency and low computational demand. They are particularly suited to bespoke (i.e. research-based) building response modelling using either modular-graphical modelling tools such as Simulink [9] – see for example [10–12], or equation-based methods such as Modelica [13] or EES [14,15].

3. Reference conduction model

A key requirement for accuracy in simplified lumped parameter building models is the correct distribution of the overall element resistance and capacitance to ensure that the element surface temperatures are accurately predicted. It is possible to attempt this analytically as has been done by Lorenz and Masy [2] and Fraisse [3] however these methods usually require complicated mathematical models and are often restricted to defined surface input excitations. In the present work, an optimisation procedure is designed to adjust the resistance and capacitance distributions so that the surface temperature of the simplified model matches that of a rigorous reference model.

The reference construction element model was created from the one-dimensional energy equation using a finite-difference scheme:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{1}$$

A full description of the discretisation and solution procedure of this equation as adopted in the present work applied to multi-layer construction elements can be found in [16]. A summary of the main discretised equations is given in the following for reference. For the temperature distribution through the body of each layer of material the following is used where the superscript n refers to the current time row and $n+1$ to the next time row:

$$T_i^{n+1} = \frac{1}{2F+1}\left(T_i^n + FT_{i-1}^{n+1} + FT_{i+1}^{n+1}\right) \tag{2}$$

in which the Fourier number, F, can be shown to be:

$$F = \alpha \frac{\Delta t}{\Delta x^2} \tag{3}$$

At the interfaces between two differing layers of material the interface temperature is obtained from the following (expressed