Energy and Buildings 42 (2010) 541–546

Contents lists available at ScienceDirect

Energy and Buildings

journal homepage: www.elsevier.com/locate/enbuild

Transient heat conduction in multi-layer walls: An efficient strategy for Laplace's method

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ARTICLE INFO

Article history:
Received 8 May 2009
Received in revised form 14 October 2009
Accepted 29 October 2009

Keywords:
Conduction transfer functions
Conduction response factors
Laplace
State Space

ABSTRACT

Enhancing load calculation tools into building simulation programs requires an in-depth revision and fine tuning of the load calculation assumptions prior to the addition of the HVAC system modelling routines. It is of special interest the analysis of transient heat conduction through multi-layer walls where, in order to improve the coupling between the passive elements of the building and the HVAC systems, an improvement of the time resolution in the calculation becomes critical. Several methods have been historically used, although recently Laplace's method has been displaced by the State Space method.

This paper proposes a new strategy for fine time resolution on the calculation of the response factors through Laplace's method considering a comparison with the performance of the State Space method when used to calculate conduction transfer functions. Our analysis shows that in order to achieve similar accuracy with both approaches, the State Space method requires significant additional computational time.

1. Introduction

The integration of HVAC systems analysis in building simulation programs requires fine time resolution. This is mainly related to response time in equipments and their control strategies, being normally constant time-steps of 1 h enough to achieve satisfactory results. Among other implications, it is necessary to modify the calculation methods used to solve the equations governing the one-dimensional transient heat conduction in walls, with variants that require shorter time steps and better time resolution.

According to Wang and Chen [1], these methods can be grouped in four categories: numerical, harmonic, response factor (RF) and conduction transfer function (CTF) methods. Numerical methods approximate the derivatives in space and time using finite difference or finite element methods. Accuracy of results, CPU time requirements and stability of these techniques depend on the number of nodes, the selected time-step and the methodology of resolution adopted. Their main advantage is a conceptual simplicity, being also relevant that both linear and nonlinear boundary conditions can be handled with this techniques.

Harmonic methods allow solving the transient conduction equations when boundary conditions can be expressed as periodic functions. Sonderegger [2] and Hittle and Pedersen [3], among others, have contributed to the development of these methods for space loads prediction.

More widely used are the RF and the CTF methods, implemented in the main simulation programs. Indeed, DOE-2 [4] computes wall conduction using the RF method; HVACSIM+ [5], TRNSYS [6] and EnergyPlus [7] use the CTF method; and BLAST [8] combines both. It is difficult, however, to undertake a comparative analysis between these methods or to evaluate their relative accuracy due to differences in their mathematical formulation and the calculation of the underlying coefficients within each methodology [9].

The RF method is based on the linear nature of the transient heat transfer equation. In general, time is divided in equal time-steps (Δt). Then time response to a unit triangle pulse, at initial time, is calculated obtaining a base of solutions. Any particular problem can be solved by a proper linear combination of this base of solutions. Thus, heat fluxes can be expressed in terms of an infinite series of the temperature history:

$$q_e(t) = \sum_{j=0}^{\infty} T_e(t-j)X(j) - \sum_{j=0}^{\infty} T_i(t-j)Y(j) \quad (1)$$

$$q_i(t) = \sum_{j=0}^{\infty} T_e(t-j)Y(j) - \sum_{j=0}^{\infty} T_i(t-j)Z(j) \quad (2)$$

Response factors for each wall are constant during the simulation [10] when they only depend on the thermal properties

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doi:10.1016/j.enbuild.2009.10.023

List of symbols

q_e	heat flux on wall external surface
q_i	heat flux on wall internal surface
T_e	external surface temperature
T_i	internal surface temperature
$XX(j)$	exterior conduction transfer function
$YY(j)$	cross-section conduction transfer function
$ZZ(j)$	interior conduction transfer function
$\Phi(j)$	flux conduction transfer function
Δt	time-step
nZ	temperature history length
nq	heat flux history length
$X(j)$	exterior response factor
$Y(j)$	cross-section response factor
$Z(j)$	interior response factor
$AA(s)$	(1,1) term in the wall's transmission matrix
$BB(s)$	(1,2) term in the wall's transmission matrix
$DD(s)$	(2,2) term in the wall's transmission matrix
L^{-1}	inverse Laplace transform
$f_\Delta(t)$	triangle function
x_m	poles of $BB(s)$
e	thickness
α	thermal diffusivity
λ	thermal conductivity

of the materials integrating the wall. Therefore, under the constant properties hypothesis, response factors will determine the transient response of an element regardless of the boundary conditions and the excitation functions that apply.

The coefficients of Eqs. (1) and (2) above, can be calculated using conventional techniques (Laplace and Z transforms [11]), by the State Space method [12] or by means of an analysis in the frequency domain through the Fourier transform [13–15].

Note that these equations involve infinite series which normally requires dealing with a large number of terms to reach sufficient accuracy. In order to overcome this and other problems of the RF method, Mitalas and Stepheson [16] proposed the CTF method, which estimates heat fluxes from short finite series,

$$q_e(t) = XX(0)T_e(t) + \sum_{j=1}^{nZ} T_e(t - j\Delta t)XX(j) - YY(0)T_i(t) - \sum_{j=1}^{nZ} T_i(t - j\Delta t)YY(j) + \sum_{j=1}^{nq} \Phi(j)q_e(t - j\Delta t) \quad (3)$$

$$q_i(t) = -ZZ(0)T_i(t) - \sum_{j=1}^{nZ} T_i(t - j\Delta t)ZZ(j) + YY(0)T_e(t) + \sum_{j=1}^{nZ} T_e(t - j\Delta t)YY(j) + \sum_{j=1}^{nq} \Phi(j)q_i(t - j\Delta t) \quad (4)$$

Basically, both methods express heat flow rate through building constructions as a function of the thermal history in both their inner and outer faces. A basic difference is that transfer functions depend on a known history of temperatures and flow rates, while response factors are based solely on the temperature history.

Among conventional methods, the Laplace transform is the most widely used, obtaining in some theoretical analysis exact results. A disadvantage is that the required roots finding procedure could result in excessive computation time and potential calculation errors. The State Space method is based on matrix algebra, it avoids searching roots and allows the treatment of multi-dimensional heat conduction. For these reasons, it has replaced conventional methods, although it could be costly in execution time when the number of nodes is increased. Frequency domain methods are proposed as a future alternative.

This paper reviews the methodologies used for the analysis of wall conduction in two internationally recognized simulation programs: DOE-2 and EnergyPlus. The first one uses the Laplace transform to calculate the response factors (from now on, Laplace's method), while the second uses the State Space method to calculate the coefficients of the conduction transfer function (or simply, State Space method). Our main objective is to compare the accuracy and speed of these methods when applied to a short time-step analysis. Results given by Laplace's method, due to its analytical nature, can be considered as the exact solution of the problem. Thus, discrepancies on the results can be taken as errors of the CTF method. First, let us briefly summarize the mathematical formulation of each method.

2. Brief mathematical description of Laplace and State Space methods

Laplace's method refers to the calculation of response factors of Eqs. (1) and (2) using the inverse Laplace transform. RF are independent of the shape that the temperature excitation functions take on both sides of the wall allowing a direct simulation of wall behaviour for any given excitation.

By definition, RF are inverse Laplace transforms of wall response to the unit triangle pulse excitation,

$$\begin{aligned} X(j) &= L^{-1}\left[\frac{DD(s)}{BB(s)}L[f_\Delta(t)]\right]_{t=j\Delta t} \\ Y(j) &= L^{-1}\left[\frac{1}{BB(s)}L[f_\Delta(t)]\right]_{t=j\Delta t} \\ Z(j) &= L^{-1}\left[\frac{-AA(s)}{BB(s)}L[f_\Delta(t)]\right]_{t=j\Delta t} \end{aligned} \quad (5)$$

where $AA(s)$, $BB(s)$ and $DD(s)$ are, respectively, the elements of the wall transmission matrix that relates surface temperatures and heat fluxes,

$$\begin{bmatrix} T_e(s) \\ q_e(s) \end{bmatrix} = \prod_{k=1}^{n} \begin{bmatrix} \cosh\left(e_k\sqrt{s/\alpha_k}\right) & \dfrac{\sinh\left(e_k\sqrt{s/\alpha_k}\right)}{\lambda_k\sqrt{s/\alpha_k}} \\ \lambda_k\sqrt{s/\alpha_k}\sinh\left(e_k\sqrt{s/\alpha_k}\right) & \cosh\left(e_k\sqrt{s/\alpha_k}\right) \end{bmatrix} \begin{bmatrix} T_i(s) \\ q_i(s) \end{bmatrix} = \begin{bmatrix} AA(s) & BB(s) \\ CC(s) & DD(s) \end{bmatrix} \begin{bmatrix} T_i(s) \\ q_i(s) \end{bmatrix} \quad (6)$$

being n the number of layers in the wall.

The unit triangle function can be written by means of three ramp functions $f_1(t)$, $f_2(t)$ and $f_3(t)$,

$$f_\Delta(t) = f_1(t) - 2f_2(t) + f_3(t) \quad (7)$$

where

$$f_1(t) = \begin{cases} 0, & t \le -\Delta t \\ (t + \Delta t)/\Delta t, & t > -\Delta t \end{cases} \quad (8)$$

$$f_2(t) = \begin{cases} 0, & t \le 0 \\ t, & t > 0 \end{cases} \quad (9)$$

$$f_3(t) = \begin{cases} 0, & t \le \Delta t \\ (t - \Delta t)/\Delta t, & t > \Delta t \end{cases} \quad (10)$$