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Structural stability analysis of single-layer reticulated shells with stochastic imperfections

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ABSTRACT

Buckling is one of the typical failure modes for single-layer reticulated shells (SLRSs). To prevent buckling, it is necessary to predict the critical buckling load and post-buckling path of the structure, and hence, structural stability analysis should be carried out. Unlike ideal structures, SLRSs with stochastic imperfections could experience different failure modes with varied post buckling paths. In the present paper, a stochastic imperfection modal superposition method is proposed for SLRS with stochastic imperfections. The SLRS structure is modeled using Timoshenko beams. Considering several possible buckling modes and random variables, Monte Carlo simulations are performed to analyze the superposition of buckling modes of the structure with stochastic imperfections. The modal combination factor is treated as a random variable and different distributions types are used for comparisons, such as uniform distribution, Gaussian distribution, T-Gaussian distribution, and triangular distribution. Based on parametric study results and comparisons with other traditional stability analysis method, the proposed method is found to provide good accuracy at considerably less computation cost.

1. Introduction

In the last few decades, with the emergence of new building materials and building techniques, long span space reticulated shells are being built wordwide. Among the different types of reticulated structures, single-layer reticulated shell (SLRS) has been used in the design for modern stadiums, planes, ships, spaceships, vehicles, and many other civil infrastructures. Buckling, which is one of the typical failure modes for SLRSs, causes embrittlement and could lead to a catastrophic disaster. However, the buckling mechanism and characteristics of SLRSs have not been sufficiently investigated [1,2]. To ensure the structural safety and avoid buckling induced disasters, it is essential to predict the critical buckling load and identify the post buckling equilibrium path of the structure. To this end, many methods, such as artificial spring method [3], displacement control method [4], arc-length method [5], and automatic incremental solution techniques [6], have been proposed. However, as the structural imperfections widely exist in structural members, structural failure modes and stability capability could be different [7,8]. Therefore, it is necessary to include the structural imperfection factor in the stability analysis methods. After the quasi-shell method was developed [7–9], Koiter first put forward an incremental theory for imperfection sensitivity analysis and the theory was later improved by Thompson and Hunt [10], Budiansky [11], and Hutchinson [12]. However, because these imperfections are applied to only a very small region of the structure, it is very difficult to apply the theory to complex structures.

In addition to analytical approaches, many deterministic numerical simulation methods, such as optimization method [13], critical imperfection modal method (CIMM) [14], and consistent imperfection modal method (CoIMM) [15] were proposed. In the CIMM, the imperfect structure is slightly different from the ideal structure, and hence it is very difficult to apply the method for large and complex structures. In the CoIMM, the lowest buckling mode is assumed to be associated with the structure displacement pattern when the structure begins to lose stability. The structural imperfection that is similar to the mode is defined as the least favorable imperfection of the structure. However, the assumption has not been validated and the lowest buckling mode might not be the least favorable imperfection. Later, another assumption that the imperfection mode is a combination of some buckling modes was proposed and validated [16]. Hence, the first 4 combination factors are calculated first using CoIMM. Then, the imperfection mode, which is related to individual or combined buckling modes, is obtained with least critical load. However, no

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random effects on the modal combination coefficient for the imperfection mode are considered.

For existing structures with stochastic defects, structural imperfections can be treated as a random variable. It was soon realized that a realistic approach to the problem could be achieved only by taking into account the inherent randomness of the imperfect geometries [17]. Based on the assumption that the initial imperfections are normally distributed, stochastic structural stability capability was predicted using Monte Carlo simulations. Later, the stochastic imperfection modal method (SIMM) was proposed and stability capability was assessed with a higher accuracy considering the randomness of the node imperfections [18,19]. However, the numbers of random variables are as high as three times of the number of nodes in the two methods mentioned earlier. As a result, the calculation cost could be very high for a complex structure with a large number of elements and nodes. In the present study, to evaluate the structural stability performance of reticulated shells with stochastic imperfections, a stochastic imperfection modal superposition method (SIMSM) is proposed for SLRSs. The SLRS structure is built using Timoshenko beams and Monte Carlo simulations are performed to include the stochastic imperfections of the structure in the stability analysis. The modal combination factor is treated as a random variable and different distribution types are used, such as uniform distribution, Gaussian distribution, T-Gaussian distribution, and triangular distribution. For several modes and random variables, the newly proposed method has better accuracy than the traditional Monte Carlo method and SIMM. Therefore, a large calculation cost does not need to be incurred for complex structures. The present paper is organized in the following sections. In Section 2, SIMSM is proposed. The probability model of the modal participation factor for reticulated shells is established based on the eigenvalue buckling analysis. Structural stability capability is analyzed by the proposed SIMSM method and Monte Carlo method. Section 3 presents a numerical example wherein the structural stability analysis is carried out for both ideal structures and structures with imperfections using the proposed SIMSM and Monte Carlo simulations. Parametric study is carried out in Section 4.

2. Stochastic imperfection modal superposition method

2.1. Stability equation for reticulated shells with imperfections

Many discretization methods, including semi-analytical and semi-discretizing method and the beam-column theory are used for structural stability analysis [9,19]. In the present study, nonlinear finite element method (FEM) and Timoshenko beam theory are used and the tangential stiffness matrix equation for a three-dimensional spatial beam can be expressed as [8,19,20]:

$$[K_T] = [K_0] + [K_L] + [K_\sigma] \tag{1}$$

where $[K_0]$ denotes linear stiffness matrix, and $[K_L]$ denotes initial displacement matrix, and $[K_\sigma]$ denotes geometrical stiffness matrix of element.

Assuming the geometric imperfection vector of structure is $\{\Delta X\}$, the new node coordinate can be updated as

$$\{X\} = \{\Delta X\} + \{X_0\} \tag{2}$$

where $\{X_0\}$ is node coordinate vector of ideal structure.

The incremental equilibrium equation can be obtained as the following:

$$[K_T]\{\Delta a\} = \{\Delta Q\} \tag{3}$$

where $\{\Delta a\}$ is the incremental displacement vector and $\{\Delta Q\}$ is the unbalanced force vector. The equation can be solved using Newton-Raphson method and arc-length method. The displacement increment of every load step and the equilibrium path in the whole loading history can be obtained.

2.2. Use of SIMSM to analyze stability

To analyze the stability of an SLRS with stochastic imperfections, the SIMSM is proposed in the present study. In the proposed method, stochastic finite element stiffness equations are built and solved by introducing a combination equation of buckling modes for the imperfection mode. At first, the eigenvalue λ_i $(i = 1, 2, \ldots, m)$ and corresponding buckling mode $\{U_i\}$ are obtained by eigenvalue buckling analysis are obtained; here, m is the number of combined modes.

To consider random geometric imperfections, the node coordinate is treated as a random variable. The randomness without adjusting the amplitude, $\{\Delta X\}'$, can be obtained as

$$\{\Delta X\}' = \sum_{i=1}^{m} (r_i\{U_i\}) \tag{4}$$

where $r_1, r_2, \ldots, r$ are the combination factors, which are assumed to be independent random variables, and $\{U_i\}$ is ith buckling mode. The combination factors are assumed to follow uniform distributions in the range of $[-1, 1]$. For each buckling mode, the displacement in the buckling modal shape of each node satisfies the following equation,

$$\max(U_{i1}, U_{i2}, \ldots U_{in}) = 1 \tag{5}$$

where n is the node number and U_{in} is the displacement of node n in the ith buckling mode.

The amplitude of $\{\Delta X\}'$ is adjusted to obtain $\{\Delta X\}$, whose amplitude is R.

$$\{\Delta X\} = R/\max\left(\Delta X'_1, \Delta X'_2, \ldots, \Delta X'_n\right)\{\Delta X\}' \tag{6}$$

Based on Eq. (2), the node coordinate vector of the imperfect structure can be obtained:

$$\{X\} = \{\Delta X\} + \{X_0\} \tag{7}$$

The stochastic finite element stiffness equation can be obtained

$$[K_T]\{\Delta a\} = \{Q\} - \{F\} \tag{8}$$

where $[K_T]$ is the stiffness matrix, $\{\Delta a\}$ is the incremental displacement vector. It is noteworthy that $[K_T]$, $\{\Delta a\}$, and $\{F\}$ are the functions of random variable r_i,

Eq. (8) can be solved by using the Monte Carlo sampling method and deterministic nonlinear finite-element analysis. The probability model of $f(r{:}\varphi)$ for the random variable $\{r\}$ can be built, where $\{\varphi\}$ is the parameter vector. After the first sampling, the vector of combination factors can be obtained:

$$\{r_{ij}\} = \{r_{1j}\} \quad (i = 1, 2, \ldots, t; \quad j = 1, 2, \ldots, m) \tag{9}$$

where r_{ij} is the stochastic combination factor of the jth mode for the ith sampling, and t is the total sampling times.

By introducing Eq. (9) into Eq. (4) and considering Eqs. (5)–(7), the node coordinate vector of imperfect structure, $\{X\}$, can be obtained. Using the updated $\{X\}$, a new model can be obtained, and Eq. (8) can be updated. By solving Eq. (8), $[K_T]$ can be obtained for every load step in the entire loading history. Now,

$$\mathrm{Det}([K_T]) = 0 \tag{10}$$

When Eq. (10) is satisfied for the first time, the structure reaches its first critical state. Thereafter, the load factor $\{q\}$ and buckling mode $\{U_{s1}\}$ can be obtained, and the critical load factor λ_{cl} is

$$\lambda_{cl}(1) = \{q\} \tag{11}$$