Engineering Structures 121 (2016) 75–84

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Effect of initial stress on attenuation zones of layered periodic foundations

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ARTICLE INFO

Article history:
Received 5 July 2015
Revised 21 April 2016
Accepted 21 April 2016
Available online 9 May 2016

Keywords:
Initial stress
Attenuation zones
Periodic foundations
Seismic isolation
Weak form quadrature element method

ABSTRACT

Layered periodic foundations have been proposed to isolate seismic waves and support upper structures. Before seismic waves arrive, initial stress exists in the layered periodic foundation due to the load from the upper structures. Based on the weak form quadrature element method, this paper proposes a novel numerical approach to study the effect of initial stress on the attenuation zones of layered periodic foundations. Comparisons with existing results in special cases are conducted to validate the proposed method, and good agreement is found. The theoretical results illustrate that the compressive initial stress shifts the attenuation zones to a lower frequency range. In addition, the maximum attenuation coefficient in each attenuation zone is not affected by the initial stress. Furthermore, frequency-domain and time-domain response analyses of a building with a finite-unit cell layered foundation are conducted. The present work is very helpful for the design and application of periodic foundations in seismic isolation.

1. Introduction

The seismic design of buildings and other structures has been the subject of research by engineers for many decades. In traditional seismic design methods, the seismic resistance of a structure is strengthened by increasing the size of the structural members. An attractive alternative can replace the traditional seismic design by using innovative methodologies, among which seismic base isolation is recognized. The seismic base isolation is a technique that mitigates the damage due to earthquakes by adding isolation bearings between the upper structure and the foundation [1]. In recent years, a number of new technologies related to seismic base isolation has been achieved [2–6].

Recently, research in the field of solid-state physics has shown that periodic structures can be designed to filter out waves with specified frequencies [7–11]. The frequency ranges where the waves are filtered are called the attenuation zones (AZs). Therefore, periodic structures have many potential application in vibration control or isolation. Guided by recent studies in the attenuation zones in solid-state physics, Shi and his co-workers proposed periodic foundations as a new method for seismic isolation [12]. Three types of periodic foundations have been studied both theoretically and experimentally till now. The first type is the layered periodic foundation, which is made of alternating layers of concrete and rubber [13]. Xiang et al. [14] experimentally investigated the isolation effectiveness of layered periodic foundation. The test results [14] show that the periodic foundation can reduce seismic response of the upper structure significantly when the main frequencies are inside the attenuation zones of the layered periodic foundation. The second type is the two-dimensional periodic foundation, where the concrete or steel cylinders coated by rubber are embedded in the concrete matrix [15]. The third type is the three-dimensional periodic foundation, where the steel cores coated by rubber are distributed in the concrete matrix [16]. Different from the traditional base isolation technique, no additional isolation bearing is needed for the periodic foundations. Previous theoretical and experimental studies show that the dynamic responses of structures with periodic foundations are significantly reduced by the periodic foundations when main frequency contents of the excitation fall into the AZs of the periodic foundations [17]. Meanwhile, periodic-material based seismic barriers surrounding buildings were proposed to block seismic waves by other researchers. Kim and Das [18,19] studied the attenuation effectiveness of periodic cylindrical shell-type waveguide composed of Helmholtz resonators. In their investigations [18,19], the seismic energy is converted into sound and heat by the periodic waveguide. Brule et al. [20] experimentally investigated wave attenuation in periodic boreholes in soil. The results in [20] show that the excitation waves in AZs will be reflected by the periodic boreholes.

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http://dx.doi.org/10.1016/j.engstruct.2016.04.049

In the researches of the traditional isolation bearings, the dynamic responses of isolation bearings have been found to be significantly affected by the axial loads. For natural rubber, high-damping rubber and lead-rubber bearings, the secant stiffness is founded to decrease with increasing the compression load [21]. Ryan et al. [22] developed a new model for lead-rubber bearings to include the effect of axial load. Similar to conventional pure concrete foundations, the periodic foundations are also designed to support heavy loads, yet the effect of axial load on the periodic foundations has not been examined. In view of the fact that stresses in the periodic foundations due to the axial loads already exist before seismic waves arrive. From hereafter the effect of axial loads is referred to the effect of initial stresses. The principal objective of this paper is to study the effect of initial stress on the AZs of layered periodic foundations, which has not been studied in previous published papers on periodic foundations.

Previous studies of the layered periodic foundations are mainly based on the plane wave expansion method (PWE) and the transfer matrix method (TMM). However, convergence of the PWE method is slow for the case where the elastic modulus of concrete is five orders of magnitude greater than that of rubber [23]. The TMM can only be used to obtain the AZs, and is unable to analyze the dynamic responses of the periodic foundations. Given the disadvantages of both the methods mentioned above, a new approach based on the weak form quadrature element method (WFQEM) is developed, which overcomes the shortcomings of the PWE and the TMM. Unlike the finite element method (FEM), no trial solutions and meshes are needed for the WFQEM [24,25]. Moreover, by employing the differential quadrature approximation (global approximation), the accuracy of the WFQEM is superior to the traditional FEM because the latter is featured with local approximation [26–28].

Though the WFQEM is well established, a lot of work is needed in order to extend the WFQEM to study the periodic foundation. First, after obtaining the stiffness and mass matrices by the WFQEM to form the governing equations including the effect of initial stress, the Bloch analysis method is employed to obtain the AZs of the layered periodic foundation. Second, to quantify the effective attenuation in the AZs, an inverse process of the procedure used for the AZs is developed. Last, the Newmark-β integration method is combined with the WFQEM to evaluate the dynamic responses of the layered periodic foundation.

In this paper, a new approach based on the WFQEM is developed for the analysis of the effect of initial stress on the layered periodic foundation. Using the WFQEM, the present authors analyzed the effect of initial stress on the attenuation zones of the pile barriers [29]. It should be noted that waves propagating in layered periodic foundations and periodic pile barriers are totally different. For layered periodic foundations, propagating plane waves are P waves and S waves. However, for pile/soil structures, propagating plane waves are in-plane waves (coupled P–SV waves) and out-of-plane waves (SH waves). Additionally, periodic pile barriers are proposed to reduce ambient vibration with frequency $f = 30–100$ Hz, while layered periodic foundations are for seismic isolation where the wave frequency mainly below 20 Hz. Moreover, periodic pile barriers can be considered as 2D periodic structures who possesses periodicity in two directions; layered periodic foundations are 1D periodic structures whose periodicity is in one direction. The remaining sections are organized as follows: The basic equations and solving procedure are given in Section 2. In Section 3, two examples in special cases are recalculated to validate the proposed method for the analysis of layered periodic foundation. In Section 4, the effects of initial stress on the width of AZs, the attenuation coefficient in the AZs and the mode shapes at the bound frequencies are discussed. In Section 5, the frequency-domain and time-domain responses of a building with layered periodic foundation are analyzed to verify the theoretical results achieved in Section 4. Some conclusions are presented in Section 6.

2. Basic equations and computational method

Fig. 1 illustrates a layered periodic foundation with initial stress, which is composed of alternating material A with thickness a_1 and material B with thickness a_2. Therefore, the periodic constant, i.e., the thickness of the unit cell, is $a = a_1 + a_2$. In the present paper, for simplicity, it is assumed that: (i) the material A and B are isotropic and linearly elastic; (ii) bonding between the interfaces of material A and B is perfect; (iii) dampings of the materials A and B are negligible; and (iv) only initial stress component σ_{zz}^0 exists and it is homogeneous in the periodic foundation. In the present paper, we focus on the effect of compressive initial stress. The sign of σ_{zz}^0 is taken to be positive for the compressive initial stress. And shear waves that propagate in the z-direction are considered.

2.1. Hamilton's principle and the WFQEM

An arbitrary layer e (material A or material B) in the z-direction can be mapped into a normalized computational domain by following coordinate transformation

$$z^{(e)} = \frac{1-\xi}{2} z_b^{(e)} + \frac{1+\xi}{2} z_t^{(e)}, \quad -1 \leqslant \xi \leqslant 1, \tag{1}$$

where $z_b^{(e)}$ and $z_t^{(e)}$ are the coordinates of the bottom and top of the layer e.

The derivatives of an arbitrary function $f(z)$ with respect to the z-coordinate can be expressed in terms of its derivatives in the ξ-coordinate

$$\frac{\partial f}{\partial \xi} = \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial \xi} = J \cdot \frac{\partial f}{\partial z}, \tag{2}$$

and

$$\frac{\partial f}{\partial z} = J^{-1} \frac{\partial f}{\partial \xi}. \tag{3}$$

where J denotes the Jacobian matrix and reduces to a scalar for the above coordinate transformation.

Let u be the displacement in the x-direction. The Green strain can be given by

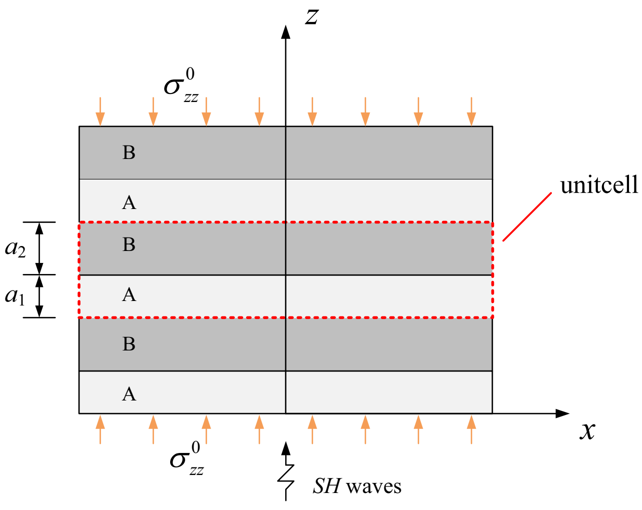

Fig. 1. Configuration of a periodic foundation with initial stress.