Engineering Structures 123 (2016) 97–107

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

A novel analytical method for the analysis of a bi-concave cable-truss footbridge

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ARTICLE INFO

Article history:
Received 14 December 2015
Revised 13 May 2016
Accepted 14 May 2016
Available online 10 June 2016

Keywords:
Bi-concave cable truss
Tensile structures
Continuous inextensible diaphragm
Closed form solution
Concentrated forces
Stays
Finite element

ABSTRACT

Bi-concave cable truss systems are not only aesthetically appealing but they also offer elegant solutions for spanning large spans required in modern buildings such as convention centres, sport arenas and bridges. These structures, however, are notoriously difficult to model correctly using non-numerical based methods. The existing analytical methods are limited to uniformly distributed loads on half or the entire span, and do not include the stays in the calculations. This paper details a novel analytical method that not only covers wide spectra of loads, including both uniformly distributed and concentrated loads on any part of the span, but also includes the stays in the calculations. The mathematical formulation was based on the fundamental assumption that the hangers form a continuous and inextensible diaphragm. The main two nonlinear equations describing the cable thrusts, unknowns of the problem, were calculated using an orthogonal displacement equation based on the boundary conditions of the cable ends at the anchorages. The predicted results are compared to finite element analysis, and good agreement was found across all the load configurations. The presented method was found to be very efficient and reliable.

1. Introduction

The use of cable systems in construction is not only aesthetically appealing but offers plenty of opportunities for innovation in the art of building. Because of their lightweight and high strength, they constitute elegant solutions for spanning large spans required in modern buildings such as convention centres, sport arenas and bridges. As the name suggests, the main structural system is the cable. With no compression or flexural stiffness, it can only be subject to tensile forces. They are referred to as tension structures, and from this characteristic stems the need to pre-tension the cable before its use, the effect of which plays an extremely important role in the stability of the structure whose own weight is particularly low. This pre-tension or preload represents the initial load that needs to be applied so that in any case of overload none of the elements becomes compressed. Because of their lightweight however one cannot ignore the susceptibility of such structures to dynamic effects. One way of addressing this shortcoming, without increasing the deck stiffness and hence the weight of the bridge, is to provide an additional inverse pre-tensioned cable under the deck. This not only enhances the bridge stiffness but also increases its load carrying ability. The cable system of interest therefore consists of a bi-concave cable truss as shown on Fig. 1. The system consists of two major cables anchored at their ends, a series of pin-ended hangers attached to the cables supporting a lightweight deck whose stiffness can be neglected.

Such a structural system is essentially discrete. It is not surprising therefore that the finite element method has been successfully used to analyse these structures [1–10]. Nonetheless, analysing these structures with the finite element method is not a straightforward process. A particular caution must be observed because of the risk of an element undergoing compression. In such a case, the stiffness in compression of the element in question must be cancelled and its load redistributed to the neighbouring ones. If the number of compressed elements is quite high, the instability of the structure becomes inevitable, and the solution process diverges. However, when the number of cables in the network becomes high, it is possible to approximate the network by a continuous system leading to analytical methods of analysis [11–23]; the spacers and the ties are replaced by a continuous inextensible diaphragm. Indeed, because they are presented in the form of exact mathematical expressions, closed form analytical solutions offer many advantages such as: a clear view into how variables, and interactions between variables, affect the result, efficiency, and

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http://dx.doi.org/10.1016/j.engstruct.2016.05.025
0141-0296/

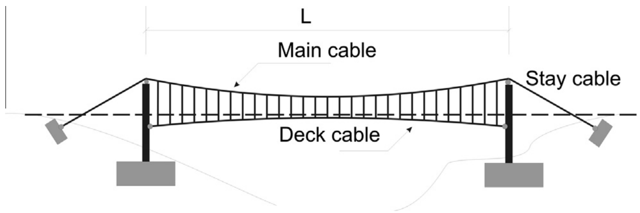

Fig. 1. Bi-concave cable truss footbridge.

hence better understanding. Most importantly, they can be used by engineers in preliminary conceptual designs for dimensioning of cable truss structures.

The development of closed form solutions is however based on very simplifying assumptions. Quite often, the second order terms in the equations of cable equilibrium and compatibility conditions are neglected to obtain linearised approximation to analyse the static equilibrium under external loads. Additional assumptions, such as the total weight of the structure being equal to zero at the time of the pre-tensioning, the absence of the stay cables, and the imposed loading being uniformly distributed of the whole or half the span, are also made, which categorically excludes the presence of concentrated loads or their combination with uniform loads. The aim of this work therefore is to develop a succinct method, which is easy to use, yet complete, sufficiently accurate, and reliable for the analysis of these structures under vertical loads. However, the following assumptions define the conditions of validity of the method:

1. the cables are perfectly flexible;
2. the hangers are inextensible;
3. the relatively tensioned cables have a sag/rise over the span of about 1/10 or less;
4. the cables are parabolic in their initial states.

2. Initial equilibrium

For the sake of clarity in the equations to follow, the indices 0 and 1 refer respectively to the main and deck cables. For example, and as shown in Fig. 2, the force H_1 refers to the pre-tension in the deck cable. Additionally, to differentiate between external loading and internally generated forces, the latter will be referred to as actions.

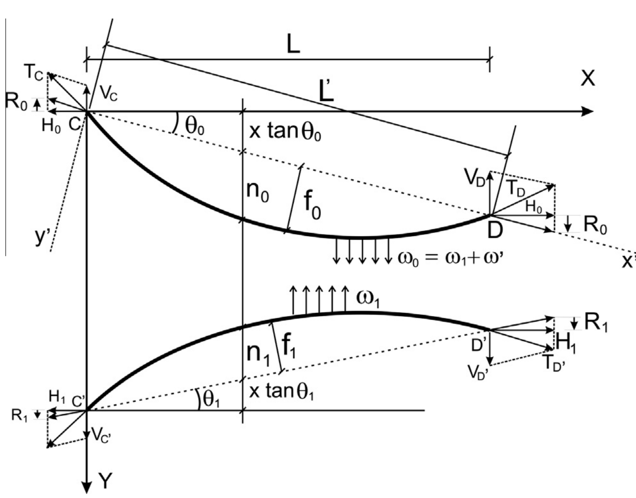

Fig. 2. Initial conditions.

To maintain equilibrium, the pre-tension H_1 in the deck cable generates a uniformly distributed action ω_1 in the diaphragm. If ω' is the weight per unit length of the cable truss, then the distributed action ω_0 acting on the main cable is given as:

$$\omega_0 = \omega_1 + \omega' \tag{1}$$

The top and bottom cables are assumed to be parabolic. In its principal axes (x', y'), the profile of the top cable is given as:

$$y' = \frac{4f_0}{(L')^2} x'(L' - x') \tag{2}$$

Introducing a change of variables, $x' = x/\cos(\theta_0)$ and $L' = L/\cos(\theta_0)$, it follows that

$$n_0 = \frac{4f_0}{L^2} x(L - x) \tag{3}$$

Similarly, the profile of the deck cable is given as:

$$n_1 = -\frac{4f_1}{L^2} x(L - x) \tag{4}$$

Consider a cut a distance x as shown in Fig. 3a.

The force $T_{C'}$ in the deck cable can be decomposed into two components: one along the vertical axis $V_{C'} = \frac{\omega_1 L}{2}$, which represents the vertical reaction due the uniformly distributed load ω_1, and another one along one of the principal axes of the parabola, which in turn is decomposed into two components H_1 and $R_1 = H_1 \tan(\theta_1)$.

Considering moment equilibrium with respect to an axis z located at a distance x, it follows:

$$\begin{aligned}\sum M_{/x} &= M(x) + \frac{\omega_1 L}{2}x - \frac{\omega_1 x^2}{2} - H_1(-n_1 + x\tan\theta_1) \\ &\quad + H_1 \tan\theta_1 x \\ &= 0\end{aligned} \tag{5}$$

Note that n_1 is by definition a negative quantity.

Taking into account the hypothesis stating that the cable is flexible and the bending moment is equal to zero at any point x, $M(x) = 0$, it follows that:

$$\frac{\omega_1 L}{2}x - \frac{\omega_1 x^2}{2} + H_1 n_1 = 0 \tag{6}$$

Introducing $\mu_1 = \frac{\omega_1 x}{2}(x - L)$, which represents the moment created by the uniformly distributed action ω_1 on a uniformly loaded and simply supported beam as shown on Fig. 3b, it follows:

$$-\mu_1 = H_1 n_1 \Rightarrow n_1 = -\frac{\mu_1}{H_1} \tag{7}$$

Using Eq. (4) yields;

$$H_1 = \frac{\omega_1 L^2}{8f_1} \tag{8}$$

Proceeding similarly for the main cable, and in virtue of Eq. (1), it follows:

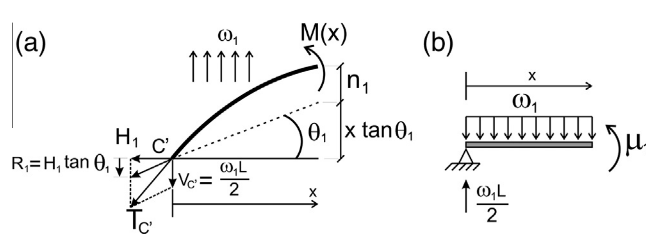

Fig. 3. Bending moment in the deck cable.