Engineering Structures 123 (2016) 313–329

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Novel design approach for the analysis of laterally unrestrained reinforced concrete slabs considering membrane action

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ARTICLE INFO

Article history:
Received 22 January 2016
Revised 3 May 2016
Accepted 17 May 2016
Available online 11 June 2016

Keywords:
Modelling
Concrete slabs
Membrane action
Failure criteria
Comparative assessment

ABSTRACT

In recent years, investigations on the behaviour and modelling of laterally unrestrained reinforced concrete slabs have been intensified due to the reserve of strength that membrane action provides. The existing analytical and numerical design approaches do not lead to satisfactory estimations, especially on failure predictions. In this paper, a novel design approach is described. The behaviour of a slab prior to the development of membrane forces is estimated through classic, renowned methods, and the vertical deflection at which membrane action begins by means of a perfect-plastic kinematic model. For larger deflections, an iterative procedure is proposed to find the distribution of membrane forces that satisfy both, equilibrium, and kinematics of the slab. Two failure criteria are included to determine the maximum load-bearing capacity and deflection. The presented approach, together with other existing design methods, are compared with 43 experimental tests of laterally unrestrained slabs conducted by different authors. The comparison illustrates a good correlation between model prediction and test results, and a clearly improved performance in terms of accuracy and precision is achieved.

1. Introduction

Membrane action and maximum capacity estimation of reinforced concrete slabs is a widely researched topic. Most of the investigations conducted so far have focused on the behaviour of laterally restrained slabs at the edges [1–4] as they require moderate deflections in order to develop compressive membrane action, which greatly enhances their load-bearing capacity. The so-called tensile membrane action, however, requires large deflections to increase the capacity of slabs with and without lateral restraint [5–7]. Investigations on the large displacement behaviour of laterally unrestrained slabs are scarce due to the absence of effective compressive membrane action.

The phenomenon of tensile membrane action in laterally unrestrained slabs was first noticed by Wood [8]. Further intensive theoretical work, experimental campaigns and development of design approaches [7,9–12] were performed during the mid-1960s. Research on this topic was abandoned, however, due to the large displacements required to initiate the development of tensile membrane forces in laterally unrestrained slabs, which significantly exceeded the serviceability limits. In recent years, the topic has gained renewed relevance as the reserve of strength that tensile membrane action provides can become relevant in the response of structures under extreme loading conditions. This includes not only fire or blast events, but also progressive collapse scenarios. This reserve of strength can play a key role within the alternative load path strategy for enhancing structural robustness in case of a column removal [13], allowing the slab to redistribute the loads to the remaining supports without collapsing. In all these extreme events, large deflections are usually tolerated as the main objective is to avoid overall structural collapse. In this direction, the investigation of laterally unrestrained slabs at large displacements has become relevant lately, in order to obtain a lower-bound estimate of the load-bearing capacity of slab floors. Several experimental campaigns were performed [14–17] and new analytical and numerical finite element-based design approaches were developed [18–22].

All these existing approaches, however, exhibit some drawbacks, making them inaccurate and less suitable for practical purposes. For this reason, a new approach for the assessment of laterally unrestrained reinforced concrete slabs was developed and is introduced hereafter.

2. Behaviour of laterally unrestrained reinforced concrete slabs

Typical specimens subjected to a static monotonically increasing uniform load show a complex behaviour [23]. At low loads,

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http://dx.doi.org/10.1016/j.engstruct.2016.05.033

Nomenclature

A	cross-sectional area per unit width
C	resultant of the compression forces in a cross-section per unit width
c	depth of the compression zone (surface to neutral axis)
$c(y)$	concrete depth distribution along the diagonal yield lines
$c(x)$	concrete depth distribution along the yield line parallel to the longer span
c_{AB}	concrete depth for simultaneous failure of concrete and reinforcement
c_{corner}	concrete depth at the corners of the slab
c_n	averaged concrete depth of the neutral axis for the ultimate pure positive bending moment ($n = 0$)
c_n'	averaged concrete depth of the neutral axis for the ultimate pure negative bending moment ($n = 0$)
d	effective depth of the reinforcement
d_{cc}	height of the axis of rotation
d_r	lever arm of the neutral axis
E_{cm}	Young's modulus for concrete
E_s	Young's modulus for reinforcing steel
$E_{s,\ eff}$	effective Young's modulus for reinforcing steel
E_{se}	elastic Young's modulus for reinforcing steel
E_{sp}	post-yielding hardening modulus for reinforcing steel
$e_x(y)$	partial elongations of the yield lines in the x-direction at the neutral axis
e_θ'	partial elongation orthogonal to the diagonal yield lines for the triangular slab regions
e_φ'	partial elongation orthogonal to the diagonal yield lines for the trapezoidal slab regions
F_C	resultant of the compression forces along the diagonal yield lines
$F_{C,c}$	resultant of the compression forces along the axial crack
F_T	resultant of the tensile forces along the diagonal yield lines
F_T'	resultant of the tensile forces along the yield line parallel to the longer span
$F_{T,c}$	resultant of the tensile forces along the axial crack
f_{cm}	average compressive cylinder strength of concrete
$f_{s,y}$	yield strength of reinforcing steel
$f_{s,u}$	ultimate tensile strength of reinforcing steel
h	slab thickness
I_{eff}	effective moment of inertia of the cross section at the corners
I_{cr}	moment of inertia of the cracked cross section
L	longer span of the rectangular slab
l	shorter span of the rectangular slab
l_b	rebar bonding length
$l_{b,y}$	rebar yielded bonding length
M_{cc}	resultant of the in-plane moments at the compression centre
m	bending moment per unit width
m_{corner}	bending moment per unit width at the corners of the slab
$m_{u,0}$	ultimate pure positive bending moment of a cross section per unit width ($n = 0$)
$m_{u,0}'$	ultimate pure negative bending moment of a cross section per unit width ($n = 0$)
m_u	positive bending moment of a cross section per unit width, caused by an ultimate curvature
m_u'	negative bending moment of a cross section per unit width, caused by an ultimate curvature
n	normal force per unit width
n_{corner}	normal force per unit width at the corners of the slab
n_u	normal force per unit width, caused by an ultimate curvature
Q_1, Q_2	statically equivalent nodal shear forces
q	applied uniform load
q_{cr}	uniform load causing first cracking
q_{lim}	uniform load causing failure
q_u	uniform ultimate flexural load given by the classical yield-line theory
q_y	uniform load causing first yielding
$s(y)$	lateral displacements along the axial crack
$s_{ini}(y)$	lateral displacements along the axial crack before membrane forces develop
$s_r(y)$	relative lateral displacements along the axial crack
$s_{y,r}$	relative displacement at the axial crack causing yielding of rebars
$s_{u,r}$	relative displacement at the axial crack causing rupture of rebars
$s_{t,b}$	average spacing between reinforcing bars in the y-direction
T	resultant of the tensile forces in a cross-section per unit width
u_b	slip of the reinforcing bar
$u(y)$	lateral displacements of the slab at the axis of rotation in the x-direction
$u_n(y)$	lateral displacements of the slab at the neutral axis for pure bending in the x-direction
V	resultant of the in-plane shear forces along diagonal yield lines
$v(x)$	lateral displacements of the slab at the axis of rotation in the y-direction
w_0	central vertical deflection
w_{cr}	central vertical deflection corresponding to first cracking
w_{ini}	central vertical deflection at which a specific point of the axial crack starts opening
w_{lim}	central vertical deflection corresponding to failure
w_u	central ultimate vertical deflection for which membrane action is assumed to begin in the new approach
w_y	central vertical deflection corresponding to first yielding
$w_{y,corner}$	central vertical deflection for which strains begin to develop at the corners
$w_{y,corner,2}$	central vertical deflection for which the cross-section completely yields
x	coordinate defining positions in the longer span direction of the slab
x_b	coordinate defining positions along the bonding length of rebars
y	coordinate defining positions in the shorter span direction of the slab
y_0	parameter defining the point of zero axial forces along diagonal yield lines
y_{cc}	parameter defining the position of the compression centre at the axial crack
y_{cr}	length of the tensile zone of the axial crack
α	angle defining the yield line pattern
β, β'	angles defining elongations of the yield lines on the horizontal plane at the axis of rotation
γ	aspect ratio of the slab (L/l)
Λ	rotation ratio (φ/θ)
ε	generic cross-sectional axial strain
$\varepsilon_{s,c}$	longitudinal strain of the reinforcing bars along the bonding length and axial crack
ε_c	concrete strain
$\varepsilon_{c,1}$	concrete strain at the maximum compressive strength
ε_{corner}	maximum compressive strains at the corners of the slab
$\varepsilon_{c,u}$	ultimate compressive concrete strain
ε_s	reinforcing steel strain