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Modified double-control form-finding analysis for suspendomes considering the construction process and the friction of cable–strut joints

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ABSTRACT

The structural configuration of slender long-span suspendomes from the construction stage to the final operational stage is critical for evaluating structural safety and resiliency. In the present study, a modified double-control form-finding (MDFF) method is proposed with consideration of the construction process and the friction of cable–strut joints based on a transient structural model of a suspendome. Based on the total Lagrangian increment formulation, the incremental equilibrium equation is built to include geometric nonlinearity. Afterwards, in the first construction step, the nodal displacement, displacement condition and the tangent stiffness matrix are built, and the finite element equations are calculated based on the existing members using the geometric nonlinear finite element method (FEM). The internal force and displacement can be obtained. In the subsequent construction stages, the nodal displacement, displacement condition and the tangent stiffness matrix are modified based on the newly added members and the friction of the cable–strut joints in present stage. Throughout the whole construction process, the inverse iteration method is used to control the structural configuration, and the initial stress increment method is used to control the cable force. After repeating the above steps, iterations are done in each key construction stage until the convergence criteria are met. The result is the final configuration of the structure including the geometry and cable forces. Based on the results of a numerical test on a suspendome, the proposed MDFF is found to be more accurate than the traditional double-control form finding method (DFF).

1. Introduction

Suspendomes, which were proposed by Kawaguchi et al. [1–3], are single-layer lattice domes (SLDs) that are stiffened with a tensegric system. Since the upper SLDs provide rigid support and reduce the flexibility of the lower tensegric system, they overcome some disadvantages of cable domes and SLDs [4]. Consequently, suspendomes have been widely used for long span public buildings such as the primary roof systems for public gymnasiums and stadiums [5,6]. However, due to their high flexibility [3], the structural configurations of suspendomes change during the construction process. Therefore, the structural performance of a suspendome during the construction stage could be completely different from that in the service stage [7]. In order to ensure structural safety, it is essential to closely monitor the structural behavior of suspendomes during the construction stage. To this end, many construction methods, including force-finding and form-finding method, were proposed to find the pre-stress control value and zero-state configuration [8,9].

In the force-finding method, the initial strain that corresponds to the designed pre-stress force is calculated, and the zero-state configuration is built to meet the designed state configuration [10]. These methods were originally proposed for cable domes prior to their adaptation to suspendomes. Based on the flexibility method, Hanaor proposed a unified method for the analysis and pre-stress design of prestressable structures [11]. Pellegrino and Calladine applied singular value decomposition (SVD) to obtain the independent self-stress modes as well as the independent displacement modes [12,13]. Considering the inherent geometric symmetry of cable domes, Yuan and Dong proposed the concept of feasible integral pre-stress modes [14] and a general method—referred to as double SVD (DSVD)—for the determination of the initial pre-stress distribution of cable domes with various forms

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[15]. Based on linear adjustment theory, a new numerical algorithm was presented for the initial pre-stress-finding procedure of complete cable–strut assemblies [16]. Employing these methods, the ideal initial pre-stress can be effectively obtained.

Besides the force-finding problems of domes, there are also form-finding problems. A narrow definition of form-finding was recently offered by Bletzinger [17] based on the former works of Lewis [18] and Haber and Abel [19]: the act of finding an equilibrium shape in a given boundary with respect to a certain stress state. In the last five decades, several methods of form-finding have been developed. It is possible to categorize these into three families: (a) stiffness matrix methods (SMMs), (b) geometric stiffness methods (GSMs), and (c) dynamic equilibrium methods (DEMs). SMMs are based on using the standard elastic and geometric stiffness matrices [20]. GSMs are material independent, having only a geometric stiffness. In several cases, starting with the force density method [21], the ratio of force-to-length is a central unit in the methodology. Several subsequent methods can be expressed as generalizations or extensions of the force density method, being independent of element type and often discussing the prescription of forces rather than force densities. DEMs solve the problem of dynamic equilibrium to arrive at a steady-state solution, which is equivalent to the static solution of static equilibrium [22].

In the construction process, the form- and force-finding problems are more complex since the configuration and force balance will change during the process and interact with each other. During the construction process, the cables are usually pre-stressed individually or in groups. The forces in the cables that are pre-stressed earlier will change after the remaining cables are pre-stressed [6,23,24]. Therefore, cable pre-stressing analysis is performed to control the pre-stress forces in cables during the construction process [25,26]. However, owing to the highly indeterminate nature of suspendomes, it is not easy to effectively control the cable configuration during the construction process. To manage such problems, a no-scaffold construction method was used for small-span structures, and several cable forces were selected as the control parameters in the analysis [27].

As mentioned above, form- and force-finding should both be considered [28]. Hence, it is crucial to track and control the force balance and configuration during the construction process for long-span pre-stressed structures [29]. Qin [30] proposed a method for controlling both the configuration and force balance of suspendomes during construction. Subsequently, Chen and Liu developed the double-control form-finding method (DFF), which considered the construction process for controlling both the configuration and force balance [31]. However, the friction of cable–strut joints was not considered in this work despite the fact that friction affects the final configuration and force balance. In contrast, the method presented in this study does consider the friction of cable–strut joints.

In the present study, a new form-finding algorithm with control of the cable configuration, cable force, and friction of cable–strut joints is proposed. First, the geometric nonlinearity is included in the FEM formula considering both the construction method and process. Then, the modified double-control form-finding (MDFF) method is proposed. Afterwards, both DFF and MDFF are applied to a numerical example of a suspendome. From the results, the errors associated with the cable force and node geometry are found to be much smaller using the proposed algorithm than with using DFF.

2. Geometric nonlinear analysis

Due to the high flexibility of suspendomes, the geometric nonlinear effect has to be considered in the analysis. Based on the total Lagrangian increment formulation (TL) [12,32], an incremental equilibrium equation considering the geometric nonlinearity was derived. Based on the principle of virtual work in TL,

$$\int_{V^e} \delta(\{\varepsilon^*\})^{\mathrm{T}}\{\sigma\}\mathrm{d}V - \delta(\{a^*\})^{\mathrm{T}}\{F\} = 0, \tag{1}$$

where $\{F\}$ denotes the equivalent nodal load vector and $\{\varepsilon^*\}$ denotes the virtual strain, which corresponds to virtual displacement vector $\{a^*\}$.

Considering the geometric nonlinearity, the strain–displacement incremental relationship is expressed as

$$\mathrm{d}\{\varepsilon\} = [\overline{B}]\mathrm{d}\{a\}, \tag{2}$$

where $\mathrm{d}\{\varepsilon\}$ and $\mathrm{d}\{a\}$ denote the strain increment and nodal displacement increment, respectively, which are the differentials of strain and displacement, and $[\overline{B}]$ is the incremental strain matrix with large deformation expressed as

$$[\overline{B}] = [B_0] + [B_L], \tag{3}$$

where $[B_0]$ is constant corresponding to the linear strain and $[B_L]$ is related to the nonlinear displacement.

Similar to Eq. (2), the virtual strain variation vector can be expressed as a function of the virtual nodal displacement variation vector

$$\delta\{\varepsilon^*\} = [\overline{B}]\delta\{a^*\}. \tag{4}$$

Substituting Eq. (4) into Eq. (1) yields

$$\int_V [\overline{B}]^{\mathrm{T}}\{\sigma\}\mathrm{d}V - \{F\} = 0. \tag{5}$$

Differentiating Eq. (5) yields

$$\int_V \mathrm{d}[\overline{B}]^{\mathrm{T}}\{\sigma\}dV + \int_V [\overline{B}]^{\mathrm{T}}\mathrm{d}\{\sigma\}dV = \mathrm{d}\{F\}. \tag{6}$$

Considering Eq. (2), the relationship between the stress increment and strain increment can be expressed as

$$\mathrm{d}\{\sigma\} = [D]\mathrm{d}\{\varepsilon\} = [D]([B_0] + [B_L])\mathrm{d}\{a\}, \tag{7}$$

where $[D]$ denotes the stress–strain relationship matrix. Therefore, the second integral on the left side of Eq. (6) can be expressed as

$$\int_V [\overline{B}]^{\mathrm{T}}\mathrm{d}\{\sigma\}\mathrm{d}V = ([K_0] + [K_L])\mathrm{d}\{a\}, \tag{8}$$

where $[K_0]$ denotes the linear stiffness matrix and $[K_L]$ denotes the initial displacement matrix:

$$[K_0] = \int_V [B_0]^{\mathrm{T}}[D][B_0]\mathrm{d}V, \tag{9}$$

$$[K_L] = \int_V [B_0]^{\mathrm{T}}[D][B_L]\mathrm{d}V + \int_V [B_L]^{\mathrm{T}}[D][B_0]\mathrm{d}V + \int_V [B_L]^{\mathrm{T}}[D][B_L]\mathrm{d}V. \tag{10}$$

The first integral on the left side of Eq. (6) can be expressed as

$$\int_V \mathrm{d}[\overline{B}]^{\mathrm{T}}\{\sigma\}\mathrm{d}V = [K_\sigma]\mathrm{d}\{a\}, \tag{11}$$

where $[K_\sigma]$ denotes the geometric stiffness matrix of the element:

$$[K_\sigma] = \int_V \mathrm{d}[B_L]^{\mathrm{T}}[D]([B_0] + [B_L])\mathrm{d}V,$$

which includes the effect of element stress for the element stiffness matrix.

According to Eqs. (9)–(11) and Eq. (6),

$$[K_{\mathrm{T}}]\mathrm{d}\{a\} = \mathrm{d}\{F\}, \tag{12}$$

where $[K_{\mathrm{T}}]$ denotes the tangent stiffness matrix:

$$[K_{\mathrm{T}}] = [K_0] + [K_L] + [K_\sigma]. \tag{13}$$