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Dynamic analysis of a thin-walled beam with open cross section subjected to dynamic loads using a high-order implicit algorithm

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ABSTRACT

In this paper, the forced nonlinear dynamic behavior of thin-walled beams with open cross section under external dynamic loads is analyzed by means of a high order implicit algorithm. This algorithm is developed using a 3*D* nonlinear model that takes into account the large torsion without any assumption on the torsion angle amplitude neither in the constitutive law nor in the derivation for governing dynamic equations. This algorithm is built by employing the following four steps: 1 – the space and time discretization procedures, 2 – a change of variable, 3 – a homotopy transformation, 4 – techniques used in the Asymptotic Numerical Method (ANM) (Cochelin et al., 2007; Mottaqui et al., 2010) [1,2]. The originality of this work reside in the fact that we use, for the first time, this algorithm for nonlinear analysis of thin-walled beams with open cross section under an arbitrary load. The space and time discretizations are performed respectively by the finite elements method and by the classical implicit Newmark scheme. The performance of the high order implicit algorithm is tested on four examples of nonlinear dynamic: a mono-symmetrical beam with a *T* cross section under external dynamic load, a mono-symmetrical beam with *U* cross-section under external dynamic load, a bi-symmetrical clamped-free beam *IPE*300 under harmonic loads and a bi-symmetrical simply supported beam with cruciform section under harmonic loads. A comparison of the obtained results with those computed by the Abaqus industrial code is given. This comparison confirms the robustness, accuracy and efficiency of this high order implicit algorithm.

1. Introduction

Thin-walled beams with open cross sections are commonly used in civil, aeronautic and mechanical engineering structures. Their structural behavior is extremely sensitive to torsional warping and local buckling, particularly when subjected to dynamic loadings at frequencies close to their natural frequencies causing nonlinear vibrations with large deformations. In addition, the coupling effects among bending, torsional, shear and axial vibrations must be taken into account in order to understand their dynamic behavior and for their safety design of these structures.

Therefore, to capture correctly the coupling between the bending, the twisting, the stretching and the large torsion of thin-walled beams with open sections under external dynamic loadings and predict accurately their dynamic responses, the required formulation may be achieved by a nonlinear dynamic model. The nonlinear dynamic of thin-walled beams with open cross section undergoing large deflections has been the subject for several research works. Several co-rotational and lagrangian finite beam formulations have been proposed in the literature [3–11]. Haijuan proposed in [12] a finite element formulation for nonlinear free vibration of thin-walled curved beams with non-symmetric open cross section and adopting a direct iteration technique to solve the nonlinear eigenvalue problem. Behiano et al. proposed in [13] a generalized beam theory (GBT) formulation to analyze the local and global dynamic behavior of open section thin-walled members under arbitrary excitation loadings based on the principle of modal superposition. Sapountzkis and Tsipiras presented in [14] a boundary element method (BEM) for evaluating the warping shear stress of bars of arbitrary nonlinear doubly symmetric constant cross-section undergoing nonuniform torsional vibration.

A finite element study was devoted in [15] to the nonlinear free torsional vibrations of thin-walled beams with bi-symmetric open cross section only in the elastic case. The solution is obtained by taking into account the linearization of the displacement field. Simo et al. [16] presented a fully nonlinear 3*D* rod model based on a exact kinematics including the effects of shear and torsional-warping deformations. The solution of this model is obtained by the finite element method. Egidio et al. [17,18]

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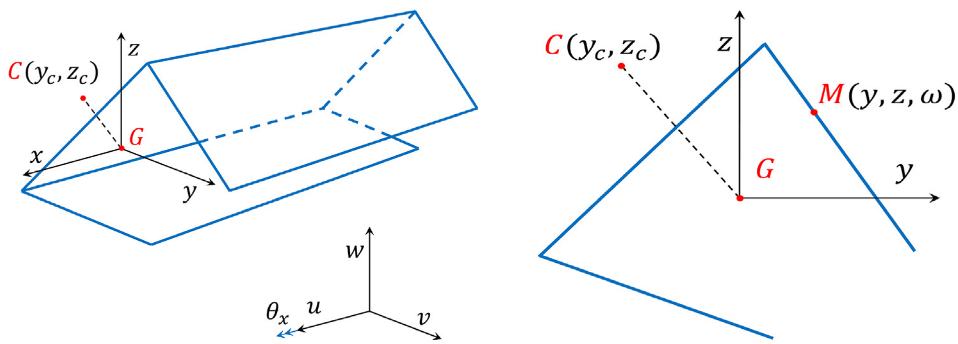

(a) Thin-walled open section beam (b) Co-ordinates of the point M on the cross section contour

Fig. 1. Thin-walled open section beam and co-ordinates of the point *M* on the cross section contour.

presented also a finite elements method solution to the nonlinear flexural–torsional vibrations of shear undeformable thin-walled open beams taking into account in-plane and out-of-plane warping by neglecting the warping inertia. This work doesn't takes into account the torsional-extensional coupling and the boundary condition are not general. Mohri et al. [19] proposed a finite elements method solution of the linear vibration problem of pre-buckled and post-buckled thin-walled open cross section beams by neglecting the warping and the axial inertia taking into account only the geometrical nonlinearity for the static loads and presenting examples of bars subjected to free vibrations and special boundary conditions.

Recently, a nonlinear model for analyzing the stability of large torsion thin-walled open cross-section is proposed in [20]. The shortening effect, pre-buckling deformation, large torsion, flexural–torsional coupling and effect of load eccentricities are taken into account in this model.

The aim of this paper is to extend this model to nonlinear dynamic problems of thin-walled beams with open cross-sections under external dynamic loadings. A high order implicit algorithm is used to investigate the dynamic behavior of these structures. This high order implicit algorithm was applied successfully to solving instationary nonlinear problem [21,22] and to structural nonlinear dynamic problems [23].

The derived dynamic equations of motion and the considered constitutive law of thin-walled beams with open section under external dynamic loads are obtained in large torsion framework without any assumption on torsion angle amplitudes. The resolution of highly coupled nonlinear dynamic equations of motion is carried out by a high order implicit algorithm. It is developed by combining: the space and time discretization procedures, a change of variable, an homotopy technique, the power series expansions and continuation methods [1,2]. The space and time discretizations are performed respectively by the finite elements method and by the classical implicit Newmark scheme. In finite element discretization procedure, a 3*D* beam element with seven degrees of freedom per node is adopted.

Several numerical examples on forced nonlinear dynamic behavior of thin-walled beams with open sections under external dynamic loads are analyzed to assess the efficiency and the reliability of the high order implicit algorithm. It is proved that this algorithm is more reliable and requires a computation time smaller than classical iterative methods.

The paper is organized as follows. In Section 2, we present the 3*D* nonlinear dynamic model of thin-walled beams with open sections and the derivation of the governing dynamic equations of motion of these structures. The Section 3 is devoted to the high order implicit algorithm. Its performance and comparisons on the illustrated typical examples will be given in Section 4. The efficiency and the concluding remarks will be summarized in conclusion given in Section 5.

2. Nonlinear dynamic model of thin-walled open cross section beams

2.1. Nonlinear kinematics

Consider a 3*D* open section straight thin-walled beam element of length L and cross section A which occupies a domain of volume Ω and the boundary $\partial\Omega$ as illustrated in Fig. 1a. The adopted reference system is $(Gxyz)$ of gravity center G and of rectangular axes Gx, Gy and Gz such that Gx is the initial longitudinal axis, Gy and Gz are the first and second principal bending axes respectively. The co-ordinates of shear center C located in Gyz plane are (y_c, z_c) and those of a point M on the section A are (y, z, ω) where ω is the sectorial co-ordinate which characterizes the warping of the section at point M for a nonuniform torsion (see Fig. 1b) [24].

In the framework of large displacements, large twist angles and small deformations, the displacements u_M, v_M and w_M of a point M on the section contour are expressed by the following nonlinear relations [25–27]:

$$\begin{aligned} u_M &= u - y((1+c)v' + w's) - z((1+c)w' - v's) - \omega\theta'_x \\ v_M &= v - (z - z_c)s + (y - y_c)c \\ w_M &= w + (y - y_c)s + (z - z_c)c \end{aligned} \tag{1}$$

where u represents the axial displacement of G, the components v and w represent the displacements of shear point C in y and z directions, θ_x is the torsional angle, ω represents the warping function and the two variables c and s are defined by the following trigonometric functions:

$$c = cos(\theta_x) - 1; \quad s = sin(\theta_x) \tag{2}$$

The symbol $(.)'$, in Eq. (1), denotes the derivation with respect to the co-ordinate x. Noting that the expressions (2) of trigonometric functions c and s are conserved in this model without any assumption on the torsion angle amplitude in both theoretical and numerical analysis. The Eq. (1) is strongly nonlinear incorporating the flexural-torsion coupling terms with the trigonometric functions c and s. The Vlasov's linear model [24] can be recovered from the Eq. (1) by approximating the trigonometric functions c and s by 0 and θ_x respectively and using linear assumptions.