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Deflection of unbonded partially prestressed concrete continuous beams

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ABSTRACT

Continuous beams are preferred to simply supported beams because of economy, fewer expansion/contraction joints and possible benefits from moment redistribution. In the design of unbonded partially prestressed concrete (UPPC) continuous beams, it is necessary to estimate their deflections under service loads in order to satisfy the requirements of serviceability limit state. A method is developed to convert the cross sectional area of unbonded prestressed tendons to the equivalent cross sectional area of non-prestressed steel. Then the moment of inertia of cracked section as well as Branson's effective moment of inertia in a UPPC continuous beam can be easily determined. The computed deflections are compared with some available experimental results, including beams with external unbonded steel tendons and those with external unbonded aramid fibre reinforced polymer tendons. The proposed method gives satisfactory predictions of deflection till the yielding of non-prestressed steel. Another equation for moment of inertia of cracked section, which was originally suggested by the precast/prestressed concrete institute (PCI) Design Handbook for bonded partially prestressed concrete beams, is also evaluated in the study. In most cases the PCI equation can also give satisfactory results but in some cases its discrepancy of deflections is larger than that of the proposed method. Compared with the method recommended in the current Chinese Code, the proposed method is applicable not only to members with the conventional high-strength steel prestressing tendons, but also to those with tendons made of other materials such as fibre-reinforced polymer.

1. Introduction

The use of unbonded tendons in prestressed concrete members not only leads to economical and simple designs, but also enables fast installation and easy replacement of defective tendons. The unbonded tendons can also provide an economic solution for strengthening and repairing existing structures. They can be used in the form of external tendons in new structures and retrofitting of concrete bridges, as well as internal tendons in beams and slabs. With the increasing use of unbonded tendons in prestressed concrete structures, there is a need for a closer examination of the design and analysis of such structures. The behaviour of prestressed concrete beams with bonded tendons is characterised by that at individual sections, as there is bonding between the tendons and the surrounding concrete. However, this is not the case for prestressed concrete beams with unbonded tendons because the tendons and the surrounding concrete generally move with respect to each other longitudinally (slip) and vertically (as a second-order effect for external unbonded tendons) [1].

In bridge construction, continuous beams are often preferred to simply supported beams as the former require fewer movement joints, provide better riding quality and allow lower maintenance cost. For the same span lengths and sections, continuous beams also have higher stiffness than simply supported beams. Previous studies of unbonded prestressed concrete (UPC) and unbonded partially prestressed concrete (UPPC) members have mainly been on the ultimate limit state in bending [2–8], but very few have addressed the behaviour at service load conditions with respect to deflection. Although numerical methods based on different theories [9–11] can be used to investigate the deformation of UPPC continuous beams at serviceability and ultimate limit states, they are inconvenient for practical designers. To determine the deflection of a UPPC continuous beam after cracking, the moment of inertia of cracked section I_{cr} is needed. Because of the lack of bond between the tendons and concrete, it is difficult to calculate I_{cr} exactly for UPPC continuous beams under specific loading. This

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Nomenclature

A_p, A_s	cross sectional areas of prestressing and non-prestressed steel respectively
A_{pe}	cross sectional area of equivalent non-prestressed steel for unbonded tendons
b, b_w	widths of flange and web respectively
c	neutral axis depth at critical section
d_p, d_s	depths to centroid of prestressing and non-prestressed steel respectively
e_0	distance between decompression F and resultant force R (=M/R)
E_c	modulus of elasticity of concrete
E_p, E_s	moduli of elasticity of prestressing and non-prestressed steel respectively
F	decompression
f_c	concrete stress in top fibre under service load
f'_c	cylinder compressive strength of concrete
f_p	stress in prestressing steel under service load
f_{pe}	effective prestress in prestressing tendon
f_r	modulus of rupture of concrete
f_s	stress in non-prestressed steel under service load
f_y	yield stress of non-prestressed steel
h_f	thickness of flange
I_{cr}	moment of inertia of cracked section
I_e	effective moment of inertia of section
I_g	gross moment of inertia of section
l	one span length in a two-span continuous beam
L	distance between end anchorages
L_0	distance between two point loads
M	applied moment at midspan sections
M_{cr}	cracking moment
R	resultant force
P	external loading at midspan sections
y	depth from top fibre to centroid of cracked section
Ω, Ω_{cr}	bond reduction coefficient at uncracked and cracked states respectively
λ	ratio of length of equivalent deformation region L_e to the neutral axis depth c at critical section
δ	deflection at midspan in an UPPC continuous beam
ρ_p,ρ_s	ratios of prestressed and non-prestressed steel respectively $\rho_p = A_p/bd_p \quad \rho_s = A_s/bd_s$

paper describes the use of a simplified method to estimate the value of I_{cr} for UPPC beams for calculation of deflections.

2. Review of previous work

There are two methods [12] to determine the short-term deflections of bonded partially prestressed concrete (PPC) beams, namely bilinear computation method and that using Branson's effective moment of inertia I_e. In the bilinear computation method, the deflection before cracking is computed using the gross moment of inertia I_g, while the additional deflection after cracking is calculated using the cracked section moment of inertia I_{cr}. The method using Branson's effective moment of inertia I_e was first applied to PPC members by Shaikh and Branson [13] in 1970. Since then, improvements have been made to the equation for I_e by different researchers. They mainly focus on: (a) the level of applied moment M at which the expression of I_e should be used; (b) the reference load or state of member deformation from which cracking moment M_{cr} is measured; and (c) the section axis about which I_{cr} is calculated. Rao and Dilger [14] compared four such methods based on I_e for their accuracy in deflection prediction, and recommended the simplified method by Shaikh and Branson [13] that gave an accurate but slightly conservative prediction. Tadros et al. [15] proposed a more rigorous and accurate method for deflection prediction by integration of curvatures at key sections along the span using the method of I_e. Scholz [16] suggested a simple method using the span-to-effective depth limits for the first-level deflection assessment of PPC members. Chern et al. [17] put forward a numerical method to evaluate the deformation of progressively cracking PPC beams. They concluded that consideration of tensile strain softening in concrete improved the predictions compared to classical theory that ignored the tensile resistance of concrete. However, the effect is not large and it mainly affects the initial post-cracking stage.

Deflection analysis of UPC or UPPC beams is more complex than for bonded tendons, as the tendon stress that is assumed constant at all sections must be determined from the deformation of the entire structure. Naaman and Alkhairi [18] proposed a method for analysis of UPC members under service load using the bond reduction coefficient, which essentially converted the UPC beams to the equivalent cases with bonded tendons, so that the previous analytical solutions for beams with bonded tendons could be used. The computation of bond reduction coefficients Ω before cracking from basic principles of mechanics is simple. However, the computation of "exact" bond reduction coefficient Ω_{cr} at the cracked state for different types of loading and tendon profiles is extremely difficult [19].

For a simply supported beam with two symmetrically disposed point loads separated by a distance, Harajli and Kanj [19] analysed the variation of Ω_{cr} versus different ratios of the total applied moment M to the cracking moment M_{cr} and different ranges of I_{cr}/I_g for beams with straight tendons. They observed that Ω_{cr} did not differ significantly from its value before cracking, and hence assumed $\Omega_{cr} = \Omega$ for practical analysis of cracked section. On the other hand, Naaman and Alkhairi [18] suggested an equation for Ω_{cr} as

$$\Omega_{cr} = \Omega \frac{I_{cr}}{I_g} \tag{1}$$

Obviously the major problem here lies in the estimation of the value of I_{cr}. Incidentally, the neutral axis location for cracked prestressed concrete beams depends not only on the geometry of the cross section and the material properties, but also on the prestressing force and the external loading. Therefore, the exact value of I_{cr} is not known until the cracked section is analysed.

Au et al. [20] extended the capability of Pannell's coefficient λ, which was the ratio of length of equivalent deformation region L_e to the neutral axis depth c at critical section, to the cracked section analysis of unbonded partially prestressed concrete members under service load. They found that, under service load after cracking of the beam and until the yielding of non-prestressed steel, λ was insensitive to the variation of the combined reinforcement index. The cubic equation established by Au et al. [20] for the neutral axis c of a cracked T-section (Fig. 1) appears as

$$c^3 + g_1 c^2 + g_2 c + g_3 = 0 \tag{2a}$$

where the constant coefficients are given as

$$g_1 = 3(e_0 - d_p) + \frac{6\lambda A_p E_p e_0}{E_c L b_w} \tag{2b}$$