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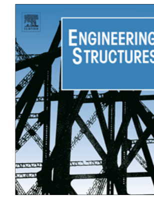

Review article

Reduced scale models based on similitude theory: A review up to 2015

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A B S T R A C T

Similitude theory is a branch of engineering science concerned with establishing the necessary and sufficient conditions of similarity among phenomena, and has been applied to different fields such as structural engineering, vibration and impact problems. Testing of sub-scale models is still nowadays a valuable design tool, helping engineers to accurately predict the behavior of oversized prototypes through scaling laws applied to the obtained experimental results.

In this manuscript it has been reviewed the developments in the methodologies used to create reduced scale models as a design tool, including those based in the use of: dimensional analysis, differential equations and energetic methods. Besides, given their importance, some major areas of research were reviewed apart: impacted structures, rapid prototyping of scale models and size effects. At last, some topics on which additional efforts can be undertaken are highlighted.

Contents

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1. Introduction

1.1. The model analysis as a simulation technique

Before production, a new design is often subjected to many investigations through theoretical analysis and experimental verification. If a new system is complex enough such that no mathematical model can be formulated to predict its behavior, even making assumptions, then extensive experimental evaluation may be necessary until the systems gains the necessary reliability and desired performance [158]. For large and oversized systems, creating the actual working conditions for testing the prototype most of the time is impossible. Even when a prototype test is possible, it is expensive, time consuming, and difficult to control [158,80].

Alternatively to the direct observation on the prototype, as well as, to the simulation techniques using mathematical models, it is possible simulate the prototype with a similar model [208,158]. This technique allows to study very complex problems with relative ease and, in many cases, with fewer assumptions that those required with the others techniques. One of the major disadvantages of similar models is that the expenses associated with experimental work must be incurred [208].

The relatively recent emergence and routine use of extremely powerful digital computer hardware and software has had a major impact on design capabilities and procedures [24]. Even so, testing of subscale models is still nowadays arguably considered, by the aeronautical engineers, as *"one of the more viable and valuable design tools since the advent of flight"* [24].

1.2. Similitude theory

Similitude theory is a branch of engineering science concerned with establishing the necessary and sufficient conditions of similarity among phenomena. It has been applied to different fields such as structural engineering, vibration and impact problems, helping engineers and scientists to accurately predict the behavior of the prototype, through scaling laws applied to the experimental results of a scale model related to the prototype by similarity conditions [177,158,142], as outlined in Fig. 1.

In mathematical terms, the *general definition of similarity* was presented by Langhaar [92] as follows:

> The function $f'$is similar to the function f, provided the ratio $f'/f$is a constant, when the functions are evaluated for homologous points and homologous times. The constant $\lambda = f'/f$is called the scale factor for the function f.

Latter, Szucs [177] stated that as long as a system can be characterized in terms of a mathematical model:

> The sufficient and necessary condition of similitude between two systems is that the mathematical model of one be related by bi-unique transformation to that of the other.

It means that, given the characteristics vectors of the prototype X_p and model X_m, containing all the parameters and variables of the systems, then a transformation matrix $[\Lambda]$ can be found, such that [177,158]:

$$\{X_p\} = [\Lambda]\{X_m\} \qquad or \qquad \{X_m\} = [\Lambda]^{-1}\{X_p\} \tag{1}$$

1.3. Objectives

The present paper aims to cover the lack of a review providing as much as possible a complete perspective of the scientific contributions to the use of scaled models based on similitude theory, ever since its establishment as a branch of the engineering science. The potential and advantages of the scale models for industry as a design tool are undeniable, but the best sources of information and knowledge to create them may not be readily available.

On the other hand, although the scientific publications usually include a brief and specific analysis of the state of the art covering the most relevant and recent researches on a specific topic, sometimes relevant contributions already presented few decades before were missed. Highly focused on the structural similitude of laminated plates and shells, Simitses et al. [162] presented the only known review. Since then, several research work were published including some innovative approaches such as the use of the governing total energy equation (Section 3.2.2.2), the use of energetic methods (Section 3.4), the sequential similitude method (Section 4.1) or even the empirical similarity methods (Section 4.2.2).

In a recent monograph by Chambers [24], NASA claims that had been leading contributions to the technology of model testing for over 80 years, since its predecessor: the NACA. Virtually every technical discipline studied by NACA and NASA for application to aerospace vehicles has used unique and specialized models, including the fields of aerodynamics, structures and materials, propulsion, and flight controls. Indeed, as claimed by Chambers [24], many of the improvements to the state of the art were presented in reports prepared under contract or in publications resulting from granted research programs, most of them totally and other partially supported by NASA. After the update to the NASA Technical Reports Server on 2013, more than fifty new reports related to scale models, structural similitude and/or scale design

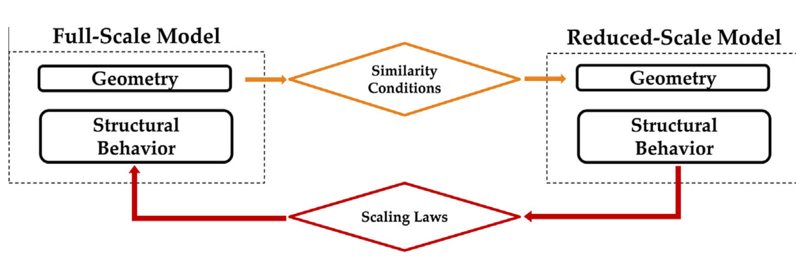

Fig. 1. Sketch for the prediction of the structural behavior of an oversized prototype, based on the experimental results of a scaled model.