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Analytical solution for layered composite beams with partial shear interaction based on Timoshenko beam theory

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The authors present an analytical solution for two-layered beam with interlayer slip. Timoshenko's kinematic assumptions are used for both layers imposing the constraint of equal cross-sectional rotation. A linear constitutive equation between the horizontal slip and the interlaminar shear force is considered. The applied loads act in the plane of symmetry of composite beam and the material and geometrical properties do not depend on the axial coordinate. Analytical solution for the deflection, slip, cross-sectional rotation and internal forces are derived. The effect of the cross-sectional shear deformation to the deflection is analyzed. Closed form solutions are given for the stresses. A Rayleigh-Betti type reciprocity relation for composite shear deformable beam with interlayer slip is formulated, whose applications are illustrated by some examples.

1. Introduction

Layered beams made of different linearly elastic materials have a wide applications in building and bridge engineering where both high strength-to-weight and stiffness-to-weight ratios are required. Two-layered composite beams are built up by two subelements of different materials and connected by shear connectors to form an interacting unit, such as timber-concrete or steel–concrete elements. The connection between the beam elements permits relative motion in axial direction which is called interlayer slip. The contact between the beam elements in normal direction is perfect. The first analytical and experimental works investigating the behavior of composite beams with imperfect shear connections can be retraced to 1950s [13,18,21,30]. The article by Newmark et al. [18] is the one of the most cited works in the area of composite beams in partial shear interaction. Newmark et al. [18] proposed a hypothesis of the linear relationship between the slip and the interlayer shear force. This assumption is based on experimental results. The static analysis done by Newmark et al. [18] used the Euler–Bernoulli beam theory and it became a basis of the subsequent investigations for layer beam systems with interlayer slip [5,7–10,12,32]. Nowadays the numerical (FEM) solutions and refining the theory of composite beams with flexible shear connections are presented by several authors. Papers [1,3,4,6,11,22,30] formulate FEM solutions for layered beams with imperfect shear connections based on the Euler–Bernoulli beam theory. Laminated beam theory with interlayer slip using the Timoshenko beam model is presented by Murakami [17]. A lot of papers deal with the FEM solution of partially connected shear deformable composite beams [15,16,19,20,24–26,33]. In paper [15] it is assumed that the cross-sectional rotation is not the same for the different beam components and the effect of shear connectors of composite beam element is described by two springs which are separately placed at the two ends of the considered beam element.

Paper by Martinelli et al. [16] gives a FEM solution for two-layer composite beams in partial shear interaction. Exact expressions of the stiffness matrix and the vector of equivalent nodal force are presented in [16]. Simple applications illustrate that the presented *exact* finite element formulation can be employed for analyzing shear-flexible steel–concrete composite beams by using only one element-per-member. This paper uses a different formulations of the governing equations which are given in [16] and the closed form formulae are derived to the deflection, slip, cross-sectional rotation and stresses. In paper [20] a FEM model based on the exact stiffness matrix for the linear static analysis of shear deformable layered beam with interlayer slip is presented. The shear connection is modeled using concentrated spring elements at each connector location. In paper by Nguyen et al. [19] the exact stiffness matrix formulation has been developed for a two-layer Timoshenko composite beam with iterlayer slip. The proposed exact stiffness matrix can be used in a displacement-based

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procedure for the static analysis of shear deformable beam with interlayer slip. Saje et al. [24] developed a finite element formulation for non-linear analysis of two-layer composite planar frames with an interlayer slip. It is assumed that the beam components obey the non-linear Reissner's beam theory. Schnabl et al. [25] propose a new locking-free strain based finite element formulation based on Timoshenko's beam theory for the linear static analysis of two-layer composite beam in partial shear interaction. A new analytical solution is presented in [26] for the analysis of the geometrically and materially linear two-layer beams with interlayer slip. The application of the proposed method is illustrated in a simply supported beam with uniform load.

This paper deals with two-layer beams with interlayer slip giving an analytical solution for the deflection, slip, cross-sectional rotation, internal forces and stresses. A slip-cross sectional rotation formulation is developed which can be considered as a generalization of slip-deflection formulation presented in [5] for Euler–Bernoulli two-layered composite beams in partial shear interaction. In the present paper the governing equations are based on Timoshenko's beam theory assuming that both layers have the same cross-sectional rotation. A linear constitutive equation between the horizontal slip and the interlaminar shear force is used. The transverse load is applied in the plane of symmetry of composite beam and the geometrical and material properties are independent of the axial coordinate.

2. Governing equations

The composite beam with partial shear interaction and its cross section are shown in Fig. 1. The cross section of beam component B_i is A_i $(i = 1, 2)$. The common boundary surface of B_1 and B_2 is $\partial B_{12} = \partial A_{12} \times (0, L)$ as illustrated in Fig. 1, where L is the length of the two-layer composite beam. The axis z is located in the E-weighted center line of the whole composite beam $B = B_1 \cup B_2$ [5]. The plane Oyz is the plane of symmetry for geometrical and material properties and loading conditions. The center of A_i is C_i $(i = 1, 2)$ and C denotes the E-weighted center of the whole cross section $A = A_1 \cup A_2$. We have [5] Fig. 1

$$c_1 = |\overrightarrow{CC_1}| = \frac{A_2 E_2}{\langle AE \rangle} c, \qquad c_2 = |\overrightarrow{CC_2}| = \frac{A_1 E_1}{\langle AE \rangle} c, \tag{1}$$

$$c = |\overrightarrow{C_1 C_2}| = c_1 + c_2, \qquad \langle AE \rangle = A_1 E_1 + A_2 E_2. \tag{2}$$

In Eqs. (1) and (2) E_i is the modulus of elasticity of beam component B_i $(i = 1, 2)$. It is assumed that both beam components follow the requirements of the Timoshenko beam theory with a common cross-sectional rotation $\phi = \phi(z)$. According to this assumption the deformed configuration of two-layer shear deformable composite beam with imperfect shear connection can be described by the next displacement field

$$\boldsymbol{u} = u\boldsymbol{e}_x + v\boldsymbol{e}_y + w\boldsymbol{e}_z \qquad (x, y, z) \in B, \tag{3}$$

where

$$u = 0, \quad v = v(z), \quad w = w_i(z) + y\phi(z) \quad (x, y, z) \in B_i \quad (i = 1, 2). \tag{4}$$

In Eq. (3) $\boldsymbol{e}_x, \boldsymbol{e}_y$ and $\boldsymbol{e}_z$ are the unit vectors of the coordinate system $Oxyz$. The interlayer slip is defined on the common boundary surface ∂B_{12} as the difference of axial components of w referring to beam components B_1 and B_2, i.e. we have (Fig. 1)

$$s(z) = w_1(z) - w_2(z), \quad (x, y, z) \in \partial B_{12}. \tag{5}$$

It is assumed that the interlayer slip is a linear function of the shear force T transmitted between the two beam components, i.e.

$$T(z) = ks(z), \tag{6}$$

where k is the slip modulus [7,8]. Application of the strain–displacement relationships of elasticity gives [29,32].

$$\varepsilon_x = \varepsilon_y = \gamma_{xy} = \gamma_{xz} = 0 \qquad (x, y, z) \in B, \tag{7}$$

$$\varepsilon_z = \frac{\mathrm{d}w_i}{\mathrm{d}z} + y\frac{\mathrm{d}\phi}{\mathrm{d}z} \qquad (x, y, z) \in B_i \qquad (i = 1, 2), \tag{8}$$

$$\gamma_{yz} = \frac{\mathrm{d}v}{\mathrm{d}z} + \phi \quad (x, y, z) \in B, \tag{9}$$

where $\varepsilon_x, \varepsilon_y, \varepsilon_z$ are the normal strains, $\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$ are the shearing strains. From the Hooke's law for the normal stress σ_z we obtain

$$\sigma_z = E_i\left(\frac{\mathrm{d}w_i}{\mathrm{d}z} + y\frac{\mathrm{d}\phi}{\mathrm{d}z}\right) \qquad (x, y, z) \in B_i \quad (i = 1, 2). \tag{10}$$

The following sub-section forces and moments are introduced

$$N_1 = \int_{A_1} \sigma_z \mathrm{d}A = E_1 A_1 \left(\frac{\mathrm{d}w_1}{\mathrm{d}z} + c_1 \frac{\mathrm{d}\phi}{\mathrm{d}z}\right), \tag{11}$$

$$N_2 = \int_{A_2} \sigma_z \mathrm{d}A = E_2 A_2 \left(\frac{\mathrm{d}w_2}{\mathrm{d}z} - c_2 \frac{\mathrm{d}\phi}{\mathrm{d}z}\right), \tag{12}$$

$$M_1 = \int_{A_1} y\sigma_z \mathrm{d}A = c_1 E_1 A_1 \frac{\mathrm{d}w_1}{\mathrm{d}z} + E_1 I_1 \frac{\mathrm{d}\phi}{\mathrm{d}z}, \tag{13}$$

$$M_2 = \int_{A_2} y\sigma_z \mathrm{d}A = -c_2 E_2 A_2 \frac{\mathrm{d}w_2}{\mathrm{d}z} + E_2 I_2 \frac{\mathrm{d}\phi}{\mathrm{d}z}. \tag{14}$$

Here,

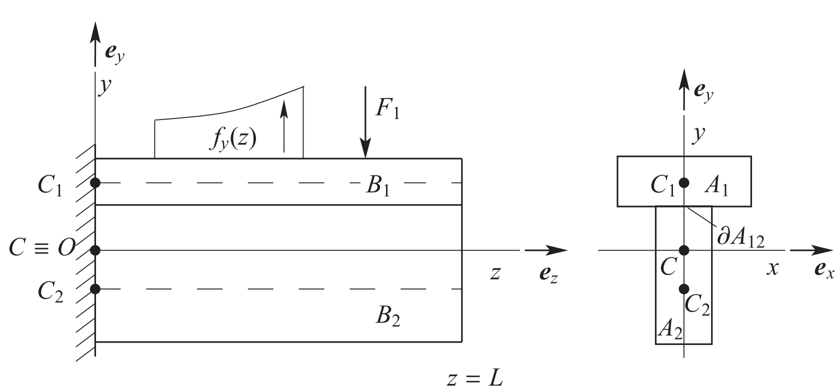

Fig. 1. Composite beam with partial shear interaction.