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Design charts for rectangular R/C columns under biaxial bending: A historical review toward a Eurocode-2 compliant update

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ABSTRACT

The objective of this paper is to review the historical context of design charts for rectangular reinforced concrete columns under biaxial bending, critically assess their evolution in time and provide a new series, compliant to the latest draft of Eurocode 2. The motivation for such a development arises from the different assumptions and recommended values prescribed in the respective National Annexes of individual countries that use the Eurocodes, which essentially yield impossible to design without the aid of specialized, typically commercial, computer codes. Along these lines, a new extensive design chart dataset of unprecedented output quality is made available herein that covers both normal and high-strength concrete as well as different steel grades. The dataset developed is fully validated against conventional procedures while the error induced in design by the use of older design aids is comparatively assessed, clearly highlighting the necessity for a new design charts dataset at least under certain loading and reinforcement configurations. The present charts are expected to provide a valuable tool for the professional community, facilitate the use of Eurocodes and minimize the epistemic uncertainly associated with the use of older or incompatible design charts in the design of reinforced concrete members.

1. Introduction

Reinforced concrete (R/C) elements are usually subjected to a combined action of biaxial bending with axial load, due to their geometry, position/orientation in the structure and, mainly, external actions. In the common case of vertical R/C elements (e.g., structural columns, bridge piers) subjected to horizontal seismic or wind loading, the above biaxial flexural stress condition at the ultimate limit state (ULS) becomes critical for their design or assessment. With the contribution of past (e.g., [1–10]) and more recent (e.g., [11–18]) research, as well as the associated software implementations (e.g., [19]), the laborious numerical solution of biaxial bending is now considered to be sufficiently well developed for use in practice. Naturally, the use of traditional design aids (in the form of tables and charts) has been gradually declined. However, there are convincing reasons to still consider design charts as a valuable tool for use in practice. Firstly, there is a number of assumptions (mostly concerning section geometry representation and material constitutive laws) as well as solution algorithms employed in commercial software that are usually obscured, i.e., treated in a black-box sense by the engineer, thus hindering the imperative need for end-user verification. In this case, validated design charts may provide a solid reference for comparison purposes. Secondly, design charts may still provide a quick and error-proof design/assessment tool for common R/C sections in office or at the construction site, especially during preliminary design stages. Last but not least, design charts remain irreplaceable for educational purposes; not only they provide tangible and quick design/assessment results in learning environments (e.g., lectures, projects and exams) but also enhance, by means of visual representation, the students' engineering perspective and judgment.

Unfortunately, when designing to the Eurocodes, it is not possible to use a uniform set of design charts for R/C members to biaxial bending among all Eurocode-complaint countries. This is due to the fact that there are specific parameters involved in the strength computation process (e.g., the reducing factor taking into account 'long term effects' on the concrete compressive strength) that are essentially country-dependent, as they take different (recommended) values according to the respective National Annexes.

Based on the above, the objective of this paper is threefold:

(a) to review the historical context of the evolution over the past decades of the design aids used for rectangular R/C columns under biaxial bending, in order to back track the missing pieces of concrete design history, hopefully providing for

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the first time the limitations of the conventional charts, highlighting the challenges to be met and identifying the emerging needs for updating the existing ones,

(b) provide a series of new generation, high quality design charts, that are automatically generated through a generic approach, according to the provisions of the latest draft of Eurocode 2 (EN1992-1-1) [20]. This extensive dataset covers both normal and high-strength concrete as well as different steel grades and is available online in the Journal website (as Supplementary Material) for use from the wider engineering scientific and professional community,

(c) validate the tool that is ad-hoc developed for the generation of the new chart dataset against older charts that reflect the assumptions made on previous design codes, while quantifying the error that is currently introduced by the use of older version design charts in the light of Eurocode 2 implementation.

2. A critical historical review

The pioneering work on biaxial design charts for rectangular R/C sections is accredited to Grasser and Linse [21,22], based on the regulations of the German Concrete Code (DIN1045) of the time [23] (Fig. 1). The stress–strain response of concrete is described by a parabolic-rectangular law with a compressive strength of β_R (based on concrete class) and a yield and crushing strain of $-2‰$ and $-3.5‰$, respectively (constant for all concrete classes). The reinforcement steel response is based on a bilinear perfectly-elastoplastic law (no hardening considered) with a tensile strength of β_S (based on steel grade), an elastic modulus of $E_e = 21{,}000\ \text{kg/mm}^2$ (=210 GPa) and a rupture strain of 5‰. The above material constitutive laws can form all potential failure states for a rectangular R/C section, depending on any arbitrary neutral axis depth and orientation. Specifically, failure can be attained either by reinforcement rupture ($\varepsilon_e = 5‰$), concrete crushing under flexure ($\varepsilon_b = -3.5‰$) or concrete crushing under prevailing compression ($\varepsilon_b = -2‰$, point A in Fig. 1), where ε_e and ε_b are steel and concrete strains, respectively. An important characteristic of the aforementioned formulation is that the section stress state was based on the obsolete *working stress* design concept i.e., no partial safety factors are pre-applied to material properties and design forces (service loads are used instead). However, at the final stage of establishing equilibrium between section resistance (calculated by stress integration) and external loading, a variable global safety factor ($v = 1.75$–2.10) equal to their ratio is imposed, depending on the resultant failure type.

The basic form of the biaxial design chart follows a Cartesian grid with axes corresponding to two perpendicular normalized moments ($m_1 > m_2$), divided into eight triangular (45°) zones corresponding to constant normalized axial force levels (n) (also called 'Rosetta-type' graphs). For a set of two external moments and an axial load, the corresponding value of reinforcement mechanical ratio (ω_0) is graphically picked from the chart, leading to the calculation of design reinforcement area (formulas will be provided later). A series of design charts [22] may correspond to different reinforcement arrangements (the most useful of which are the distributed and the 4-corner-bar types), different cover-to-dimension ratios (i.e., reinforcement position) and different steel grades. An important note is that since (a) normalized values are applied and (b) yield/crushing concrete strains are constant, every design graph is applicable to any concrete class. On the contrary, different charts should be provided for different steel grades, since steel yield strain (β_S/E_e) depends on steel tensile strength.

With the advent of the *ultimate limit state* (ULS) design philosophy, the DIN1045-based design charts had to be revised, since the aforementioned global safety factor (v) needed to be replaced with partial safety factors for material properties and external actions. Therefore, about 10 years later, a new set of design charts was issued by CEB/FIP, under the chairmanship of Prof. E. Grasser [24] (Fig. 2). As far as the material properties are concerned, β_R and β_S were replaced by $f_{cd} = f_{ck}/\gamma_c$ and $f_{yd} = f_{yk}/\gamma_s$ for concrete and steel, respectively, where f_{cd}, f_{yd} are design strengths, f_{ck}, f_{yk} are their characteristic counterparts and γ_c, γ_s are partial safety factors. The response curves for both materials were unaltered, with only steel ultimate strain doubled to 10‰. The potential ultimate limit (failure) states were modified accordingly, producing a new series of biaxial design charts that were broadly utilized until recently, even if slight code modifications occurred in the meantime, particularly due to advances in steel technology (e.g., steel ultimate strain further increased to 20‰ in [25]). A meticulous comparison between DIN1045 (Fig. 1) and CEB/FIP (Fig. 2) charts shows that for – approximately – the same chart type and for the same external loading, the former yields considerably larger reinforcement mechanical ratios; this is mainly attributed to the different design philosophy i.e., service vs. ultimate load input, which justifies the above revision.

The recent unification of structural design national codes of EU countries under the new set of Eurocode drafts has brought a few modifications which render the previous CEB/FIP charts unsuitable, under certain conditions (Fig. 3). The most important difference is that Eurocode 2 (EN1992-1-1) [20] specifies concrete design strength as $f_{cd} = \alpha_{cc} \cdot f_{ck}/\gamma_c$, where α_{cc} (=0.8–1.0) is a reducing factor taking into account '*long term effects on the compressive strength and unfavorable effects resulting from the way the load is applied*', with a recommended value of unity and open to regulation through each country's National Annex. On the contrary, CEB/FIB charts had already considered a fixed factor equal to 0.85 for the above effects, which was *hardcoded* in the stress integration procedure as a post-operator to concrete stress ($0.85 \cdot \sigma_{cd}$, see Fig. 2). As a result, this factor cannot be deliberately modified if needed (e.g., the recommended value is 1.0 in all countries, except for Germany, Greece, Italy and Belgium where it remains equal to 0.85). Another difference is that the perfectly-elastoplastic stress–strain bilinear law for steel now has no upper strain limit ε_{ud} (compared to the 10‰ limit in CEB/FIB charts) and becomes finite only if strain hardening (k) is considered. Consequently, for unlimited steel strain, the section ULS is always determined by concrete crushing, either under flexure or prevailing compression. Finally, the formerly established parabolic-rectangular stress–strain relationship for concrete with yield and ultimate strains (ε_{c2}, ε_{cu2}) equal to $-2‰$ and $-3.5‰$, respectively, ceases to apply for high strength concrete (from class C55/67 and above), where modified values for ε_{c2}, ε_{cu2} and n (polynomial order of the ascending branch, previously constant: $n = 2$ for parabolic curve) are recommended (Table 3.1 in [20]). It is also noted that Eurocode 2 [20] also provides an alternative concrete constitutive law that exhibits the experimentally observed strain softening of concrete under compression (easily handled by the employed numerical scheme), however, it is currently recommended only for nonlinear structural analysis and not for section design. The above significant novelties require the drafting of a new series of Eurocode-2 compliant design charts, which will be discussed in the subsequent sections.

It has to be noted here that apart from the DIN1045- and CEB-based design charts discussed above, there are also a few other national documents that provide similar design aids, such as the British Standards Institution (BS) [26] and American Concrete Institute (ACI) [27]. The main difference is, however, that the above documents only provide design charts for *uniaxial* bending with axial load without explicitly handing the biaxial bending case; though, an implicit and rather complicated scheme (reciprocal load or load contour method) is recommended by the ACI provisions using approximate modification factors in order to reduce the biaxial problem to its uniaxial equivalent.