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Assessment of epistemic uncertainties in the shear strength of slender reinforced concrete beams

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ABSTRACT

Selection of an appropriate model for shear capacity of reinforced concrete (RC) slender beams poses a number of challenges. First, different models with different levels of conservatism have been proposed in an attempt to describe shear resistance. Second, according to Reineck et al. (2014), code provisions for shear capacity of RC beams with shear reinforcement have been primarily derived from test data with respect to the required amount of shear reinforcement and the calculation of maximum shear capacity. Third, current models have been developed based on databases presenting two major drawbacks: (i) most data points are crowded in the small size range, and (ii) the means of the subsidiary influencing parameters are very different within different intervals of beam size (or beam depth). In this study, a filtered database is used in such a way to circumvent the drawbacks mentioned above. A random variable “model error”, i.e. ratio experimental to predicted shear strength, is associated to each of the shear models analyzed in this work (NBR 6118, ACI 318, EUROCODE 2, and CSA A.23.3). It was observed that in some cases, most notably for the effective depth, a trend exists for a decrease in the “model error” as the effective depth increases. Considering the limitations of the four analyzed models, a nonlinear regression model was proposed. The database presented by Reineck et al. (2014) was used in the assessment of the effectiveness and accuracy of the proposed regression model. No trend was found associated to the most significant variables in the shear strength prediction, i.e. a uniform level of conservatism is attained throughout the range of these variables. The regression model proposed herein and the attendant statistics of the model error (mean, coefficient of variation and type of distribution) can be easily used in a reliability analysis procedure to assess safety levels implicit in different design procedures.

1. Introduction

The selection of an appropriate model for shear capacity of reinforced concrete (RC) slender beams poses a number of challenges. First, different models, e.g. NBR 6118 [1], ACI 318 [2], EUROCODE 2 [3], CSA A.23.3 [4], have been proposed in an attempt to describe beam resistance to shear, with different levels of conservatism for each model [5]. Second, according to Reineck et al. [6], code provisions for shear capacity of RC beams with shear reinforcement have been primarily derived from test data with respect to the required amount of shear reinforcement and the calculation of maximum shear capacity. Third, current models have been developed based on databases presenting, according to Bazant & Yu [7], two major drawbacks: (i) most data points are crowded in the small size range, and (ii) the means of the subsidiary influencing parameters, such as the steel ratio and shear-span ratio are very different within different intervals of beam size (or beam depth). Neglecting or underestimating these uncertainties may estimate incorrectly the corresponding probabilities of failure thus compromising safety of RC structures subjected to shear.

Epistemic uncertainties are related to limited knowledge. As pointed out by Melchers [8] epistemic uncertainties refer to those that might be reduced with: (i) additional data or information, (ii) better modeling, and (iii) better parameter estimation. All these topics are dealt with in the research presented herein.

In this study, the database assembled by Ribeiro [9], encompassing experimental results for the shear resistance of slender beams with stirrups has been enlarged. A random variable “model error” (epistemic uncertainty) is associated to each of the shear models analyzed in this work. Better estimates of the statistics of the “model error” are obtained by the use of a filtered database in such a way to circumvent the aforementioned drawbacks. A multi-regression analysis is performed in order to derive an expression that best describes shear resistance of RC slender

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beams to be used in conjunction with reliability analyses and to identify the most significant parameters in the proposed equation for shear resistance. To this end, a model that provides a smaller coefficient of variation and the same level of conservatism with respect to the main variables affecting shear capacity of RC beams is sought. The information derived herein can be easily used in the estimation of the shear strength capacity of RC beams and the attendant model error, which in turn is needed in the assessment of the corresponding implicit reliability levels of a given design recommendation.

2. Shear models

Different procedures exist for the computation of the shear capacity (τ_{calc}) of RC beams. These models provide shear capacity as the sum of the concrete and steel stirrups contributions, τ_c, and τ_{sw}, respectively:

$$\tau_{calc} = \tau_c + \tau_{sw} \tag{1}$$

In Eq. (1), the first term, τ_c, is intended to account not only for the concrete shear strength in the uncracked concrete beyond the end of the inclined cracks but also for the complementary truss mechanisms, namely aggregate interlock along a diagonal crack and dowel action provided by the longitudinal reinforcement [6,10,11].

The contributions, τ_c, and τ_{sw}, according to model I of NBR 6118 [1], the simplified model from ACI 318 [2], the model in EUROCODE 2 [3] and the simplified model from CSA A.23.3 [4] are shown in Tables 1 and 2, respectively. In these tables, $\boldsymbol{f_c}$ stands for the concrete compressive strength, $\boldsymbol{\rho_w}$ for stirrups ratio, $\boldsymbol{f_y}$ for steel yield strength, and θ for the angle of the concrete struts.

As can be seen from those tables, these models differ in the values set for both the concrete and stirrups contributions. For the design of new structures, object of this study, EUROCODE 2 completely neglects the concrete contribution, τ_c. On the other hand, in the evaluation of the concrete contribution τ_c, ACI 318, CSA A.23.3 and NBR 6118 codes rely only on the concrete strength, ignoring the aggregate interlock along a diagonal crack and the dowel action provided by the longitudinal reinforcement. It is worth mentioning that each code has a limit value of the concrete compressive strength: for CSA A.23.3 and ACI 318 the upper values are 60 and 70 MPa respectively, while EUROCODE 2 and NBR 6118 allow values up to 90 MPa.

Taylor [12] and Kani [13] have shown that the shear strength of concrete beams decreases as the depth of the beams increases. Sneed and Ramirez [14] have stated that the reduction in the shear with increasing effective depth in beams is influenced not only by size effects but also by differences in behavior and mode of shear transfer at failure (beam action versus arch action). More recently, Mari et al. [15] have also presented a mechanical based shear model that includes size effects. All the above RC design codes do not consider these findings.

With respect to the stirrups contribution, τ_{sw}, both the CSA A.23.3 and the EUROCODE 2 use the same expression; however, EUROCODE 2 allows the designer select different values of the strut inclination θ in the range between 21.8° and 45° while CSA A.23.3 sets θ equals to 35°. A smaller value of θ allows, according to Eurocode's equation, a reduction in the amount of shear reinforcement ratio necessary to achieve the same τ_{sw}.

Table 1
Shear models for concrete contribution.

τ_c (MPa)
NBR 6118 (2014) – Model I $f_{ck} \leqslant 90$ MPa $\theta = 45^\circ$
$\tau_c = \tau_{c0} = 0.126(f_{ck})^{2/3}$
ACI 318 (2008) – $f'_c < 70$ MPa $\theta = 45^\circ$
$\tau_c = 0.17\sqrt{f'_c}$
EUROCODE 2 (2004) – $f_{ck} \leqslant 90$ MPa $21.8^\circ \leqslant \theta \leqslant 45^\circ$
$\tau_c = 0$
CSA A23.3 (2003) – $f'_c < 60$ MPa $\theta = 35^\circ$ $\beta = 0.18$
$\tau_c = 0.9\beta\sqrt{f'_c}$

3. Shear database

3.1. Ribeiro and Calixto [5]

Ribeiro and Calixto [5] have assembled a shear database comprising results from 265 tests of beams with stirrups and diagonal tension failure. The percentage of beams with effective depth larger than 60 cm was equal to 10%. The shear models considered in that study are models I from NBR 6118 [1], the simplified model from ACI 318 [2], and the model from EUROCODE 2 [3]. That study also presented details of the beams including beam geometry, steel area (both for longitudinal steel and stirrups), concrete compressive strength, steel yield strength, and rupture stress. Ribeiro and Calixto observe the paucity of experimental results for beams with effective depth larger than 60 cm. It was also observed a general trend for the models analyzed for non-conservative results for beam depth in this range.

3.2. Enlarged database

Following the need identified by not only Ribeiro and Calixto [5] but also by Bazant & Yu [7], in this study, a literature survey was performed in order to increase the available database, especially with respect to larger beam depths. This effort resulted in the addition of 15 RC beams with stirrups to the Ribeiro and Calixto database, comprising experimental results from Collins & Kuchma [16], Zararis [17], Sherwood et al. [18], Ghannoum [19], and Yoshida [20].

Regarding beams with stirrups, – the ones with practical interest –, after discarding few experimental data for not providing enough required information, it resulted in 273 RC beams. This enlarged database is shown in the Appendix. In this database, only beams with vertical stirrups and subjected to point loads have been selected. Results corresponding to beams with stirrups having steel yield strength above 950 MPa have also been ignored, since this does not correspond to cases of practical interest. All beams have been made of normal-weight concrete and have failed due to diagonal tension. They have ratio shear span to effective depth, $\boldsymbol{a/d}$, greater than 2.5, and aspect ratio (width to effective depth), $\boldsymbol{b_w/d}$, with an upper limit equal to 5. The statistics of the most influential variables of this enlarged database is presented in Table 3.

3.3. Filtered database

As pointed out by Bazant & Yu [7], current databases of shear test results present two major drawbacks: (i) most data points are crowded in the small size range, and (ii) the means of the subsidiary influencing parameters, such as the longitudinal steel ratio, $\boldsymbol{\rho_w f_y}$ and $\boldsymbol{a/d}$ ratio, are very different within different intervals of beam size (or beam depth). This problem has also been observed in the enlarged database of beams with stirrups when it was split in three ranges of the effective depth $\boldsymbol{d}$ ($\boldsymbol{d} < 30$ cm, 30 cm $\leqslant \boldsymbol{d} < 60$ cm, and 60 cm $\leqslant \boldsymbol{d}$) as shown in Table 3.

In this study, a filtered database was carefully assembled in such a way to attempt to circumvent the drawbacks mentioned above. In this filtered database the most important variables $\boldsymbol{f_c}$, $\boldsymbol{\rho_w f_y}$, $\boldsymbol{\rho_l}$, and $\boldsymbol{a/d}$ would have similar averages within each effective depth interval. As all the beams with effective depth $\boldsymbol{d} \geqslant 60$ cm did not have $\boldsymbol{\rho_l} > 4\%$ and $\boldsymbol{\rho_w f_y} > 0.2$ kN/cm^2, beams with these values of longitudinal steel ratio and stirrup strength were at first dis-