Engineering Structures 114 (2016) 25–34

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Probabilistic seismic demand model and fragility estimates for rocking symmetric blocks

Esmaeel Bakhtiary [a,*], Paolo Gardoni [b]

[a] School of Civil and Environmental Engineering, Georgia Institute of Technology, Mason Building, 790 Atlantic Drive, Atlanta, GA 30332-0355, United States
[b] Department of Civil and Environmental Engineering, MAE Center: Creating a Multi-hazard Approach to Engineering, University of Illinois at Urbana-Champaign, 3118 Newmark Civil Engineering Laboratory, 205 N. Mathews Ave., Urbana, IL 61801, United States

ARTICLE INFO

Article history:
Received 2 April 2015
Revised 13 December 2015
Accepted 26 January 2016
Available online 18 February 2016

Keywords:
Rocking bodies
Symmetric blocks
Overturning probability
Earthquake excitation
Bayesian approach
Fragility curves
Maximum likelihood

ABSTRACT

This paper presents a probability model to predict the maximum rotation of rocking bodies exposed to seismic excitations given particular earthquake intensity measures. After obtaining the nonlinear equations of motion and a clarification of the boundaries applied to a rocking body needed to avoid sliding, a complete discussion is provided for the estimation of the approximate period and equivalent damping ratio for the rocking motion. After that, instead of using an iterative solution, which has been proven defective, a new approximate technique is developed by finding the best representative ground motion intensities. Suitable transformations and normalizations are applied to these intensities, and the Bayesian updating approach is employed to construct a probability model. The proposed probability model is capable of accurately predicting the maximum rotation of a symmetric rocking block given the displacement design spectra, peak ground acceleration, peak ground velocity, and arias intensity of an earthquake. This probabilistic model along with the approximate capacity of rocking blocks are used to estimate the fragility curves for rocking blocks with specific geometrical parameters. In the end, a comprehensive and practical form of fragility curves are provided for design purposes along with numerical examples.

1. Introduction

Freestanding, non-structural components, like building components or unanchored structures, rely on their self-weight (gravity force) for stability from overturning. These structures include statues in museums, electrical equipment, and sensitive devices in hospitals. In many cases, the buildings, in which these components are placed, do not fail during an earthquake; however, failure of these freestanding components might cause enormous financial loss and fatal accidents. Therefore, in recent years, the attention of various researchers has focused on the rocking response of freestanding rigid blocks. Early studies on the rocking response of a rigid block supported on a base undergoing horizontal motion was presented in [1,2]. Current investigations use the formulation of the rocking motion derived in these studies. These models have been used to describe the behavior and the overturning of rocking bodies subject to base excitations. Some researchers employed harmonic loading (e.g., [3–6]), and many others used earthquake ground motions (e.g., [7–9]). In Ref. [1] overturning criteria have been suggested for freestanding blocks exposed to selected ground excitations. The overturning probability increases as the amplitude of the peak ground acceleration (PGA) increases [9]. Moreover, the fragility of rocking bodies increases as their slenderness increases or their size decreases [9]. Some new studies also have focused on the mitigation of rocking bodies using base isolation. An updated formulation of motion usable for base-isolated blocks can be found in the literature ([10–13]). These studies also showed the effectiveness of an isolation system.

A probabilistic approach to determine the overturning probability of blocks exposed to earthquake excitations has been premiered in [9]. After that, other studies also focused on finding the overturning probability of blocks. In particular, [14] undertook a probabilistic investigation and found relationships for the overturning fragilities as a function of peak ground acceleration (PGA), peak ground velocity over peak ground acceleration (PGV/PGA) and spectral acceleration (S_a). These results were checked with experimental data in [15] to show that the fragilities are consistent with the experimental results. Recently, it was shown that the fragility of rocking bodies has a strong correlation with the peak displacement demand (PDD) of an earthquake [16]. Before these studies,

* Corresponding author. Tel.: +1 (217) 333 5103.
E-mail addresses: esmaeel@gatech.edu (E. Bakhtiary), gardoni@illinois.edu (P. Gardoni).

http://dx.doi.org/10.1016/j.engstruct.2016.01.050
0141-0296/

an analytical solution for the free vibration period of rocking bodies, which relates the period of motion T to the angle of rotation θ had been developed in [1]. The rocking motion of free bodies with a single degree of freedom system was estimated, and an approximate damping ratio for rocking blocks was suggested [17]. The availability of period and the damping ratio for rocking structures in association with response spectra motivated researchers [17] to propose a relatively simple procedure to estimate the maximum rotation of freestanding bodies exposed to ground motions. An updated version of this process also has been used in the Federal Emergency Management Agency recommendations. However, a comprehensive investigation of this method was conducted in [18], and it was shown that this iterative method was not accurate and should be abandoned.

This paper develops a comprehensive model to compute the maximum rotation of rocking bodies considering 200 different ground motions. Using these data, a Bayesian framework is employed to develop a probabilistic demand model for the rocking motion of symmetric rigid blocks. This model properly accounts for the prevailing uncertainties, including model errors arising from an inaccurate model form or missing variables, measurement errors, and statistical uncertainties. To achieve a better approximation of ground motion intensities usable for rocking bodies, the formula for the period of motion developed in [1] is used, and an approximation for the damping ratio is developed. The period and damping ratio are used to normalize the ground motion intensities. Instead of restricting the solution to specific ground motion intensities, this paper uses an initial set of 23 different ground motion parameters to investigate their effect on the maximum rotation of rocking bodies. The relevant ground motion parameters are then selected using the generated data. After that, using the developed probabilistic model, fragility curves are generated. The fragility curves are defined as the conditional probability of overturning of a rocking body for a given set of geometrical properties and earthquake intensities. To better understand the influence of geometry on the probability of overturning, first a comparison between fragility curves for blocks with a constant slenderness and different size is presented, and after that, the same comparison is carried out for blocks with a constant size and different slenderness. These comparisons are used to obtain a comprehensive set of fragility curves. This set is valid for a broad range of the slenderness and size of blocks. At the end of the paper, some examples are solved to clarify the use of the fragility curves, and conclusion remarks are provided.

2. Dynamics of rocking bodies

The purpose of this section is to construct suitable nonlinear equations of the rocking motion of rigid symmetric bodies exposed to earthquake excitations. There are various approaches in the literature to construct the equations of motion and consider the loss of energy during an impact. The conservation of angular momentum [19] is used in this paper. Moreover, the criteria required to initiate the rocking motion and the maximum threshold for collapse or overturning of the block are discussed.

2.1. Nonlinear equations of motion

All geometrical parameters that characterize the rigid body are shown in Fig. 1. Where, C is the center of mass of the body, r is the distance from C to the corner of rotation, θ is the angle of rotation of the body, h is the vertical height of the center of mass, b is the horizontal distance from the center of mass to the corner of rotation, and α is the angle between the line connecting one corner to the center of mass and the vertical line. The equations of motion

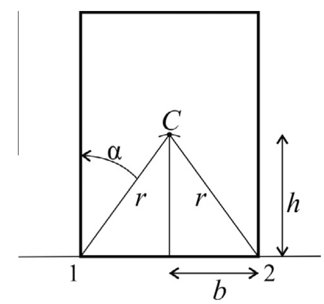

Fig. 1. Geometrical parameters of the rigid block.

are obtained by evaluating the equilibrium of all forces acting on the system during the rocking motion. Forces acting on the body during the rocking motion are shown in Fig. 2 for both rocking around the left corner (Corner 1) and the right corner (Corner 2). The equation of the movement for this single degree of freedom system can be obtained by writing the equilibrium of moments around the corner of rotation. In these figures, θ is the angle of rotation of the body, and $\ddot{x}_g$ and $\ddot{y}_g$ are the horizontal and vertical ground motion accelerations, respectively. Positive directions for ground displacements are rightward and upward for the horizontal and vertical motions, respectively. The same positive directions are assumed for velocities and accelerations. The rotation θ is positive when the body rocks counterclockwise and is negative when the body rocks clockwise. First, the counterclockwise rotation is considered, and equations can be obtained in a similar way for the clockwise rotation. For the counterclockwise rotation, the equilibrium of moments reads

$$I\ddot{\theta} - m\ddot{x}_g r\cos(\alpha-\theta) + m(g+\ddot{y}_g)r\sin(\alpha-\theta) = 0 \tag{1}$$

where $I = I_C + mr^2$ is the moment of inertia relative to the corner of rotation, I_C is the moment of inertia relative to the center of mass C, and g is the gravitational acceleration. In Fig. 2a, the term $mr\dot{\theta}^2$ is the centrifugal force acting on the body due to the angular velocity possessed by the body. However, because it is acting toward the corner of rotation, the resultant moment of it is zero, and it does not appear in the equation of motion. The term $mr\ddot{\theta}$ is the translational inertia force due to the rotational acceleration $\ddot{\theta}$.

The equation of motion for the clockwise rotation is obtained similarly by taking into account that, in this case, a rotation associated with a lifting of the base is negative. The equilibrium for the clockwise rotation is written by referring to the forces reported in Fig. 2b. This equation reads

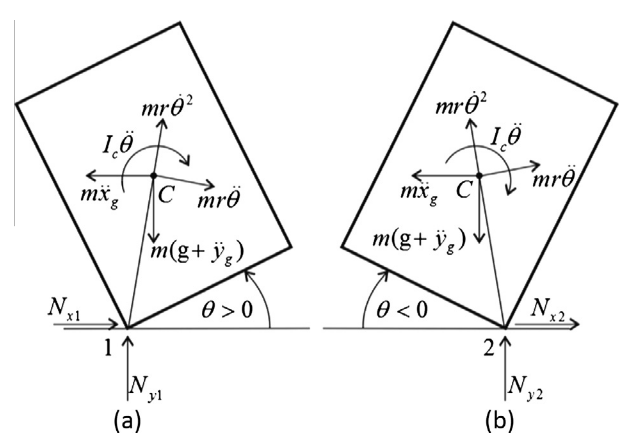

Fig. 2. Forces acting on the rigid body for (a) counterclockwise rotation and (b) clockwise rotation.