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The effect of interference of corrosion pits on the failure probability of a reinforced concrete beam

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ABSTRACT

Assuming given corrosion states this paper studies the effect of interference of corrosion pits on adjacent rebars on the probability of bending failure of a corroded reinforced concrete beam. Spatial distribution of localised corrosion along a beam is considered in the analyses. The probability of failure is estimated using Monte Carlo simulation. Uncertainties in material properties, geometry, loads, and corrosion damage are taken into account. Statistical data for the extent and location of corrosion is taken from literature and utilized in order to calibrate parameters of corresponding probability distribution functions. Results show that considering possible interference of localised corrosion has substantial influence on the estimated probability of failure. The results are compared with results from two practical methods which appeared not always to be conservative.

1. Introduction

The reliability of reinforced concrete (RC) structures is dependent on applied loads and the remaining strength of the structural elements, which is affected by the degree of damage. Corrosion of RC structures is recognized as a major deterioration cause. Laboratory and field observations of deteriorated RC structures show that corrosion damage in structural elements exposed to chlorides is not homogenous, but spatially variable along and around the rebars [1,2].

In the recent years, there has been considerable effort in the reliability assessment of RC members with corroding rebars [3–12]. In general, these probabilistic assessments have focused on spatial variability of the localised corrosion based on sectional analysis where the effect of corrosion is accounted for in individual cross-sectional areas. While much work has progressed to assess the effect of spatial variation of localised corrosion on the reliability of RC beams, there appears to be no work considering possible interference of localised corrosion on adjacent rebars.

The impact on the bending ultimate capacity (M_{ult}) of possible interference of corrosion pits on adjacent rebars was quantified by Kioumarsi et al. [13–15]. These studies were based on 3D finite element modelling. Interference of corrosion pits was explained by coalescing crack planes which eventually might bridge pits on adjacent bars.

The present paper studies the effect of the spatial variation of localised reinforcement corrosion on the structural reliability of an under-reinforced concrete beam. Particular emphasis is placed on the interference of localised corrosion on adjacent tensile rebars. The paper describes the impact of given corrosion states. The mechanism and development of reinforcement corrosion are out of the scope of the paper. Under-reinforced beams are designed such that they show a ductile failure mode. Probability of failure is estimated using Monte Carlo simulation. Uncertainties in material properties, geometry, loads, corrosion damage are taken into account. Statistical distribution functions to describe the location and size of the pits along a corroded rebar are not available. To this end, statistical data for the size and location of corrosion is taken from literature and utilized in order to calibrate parameters of corresponding probability distribution functions.

The paper continues with a description of a deterministic modelling approach for the residual capacity of concrete structural elements with emphasis on modelling interference of corrosion pits for a selected case study. Section 3 then introduces distribution functions for the spatial variation of corrosion along a rebar. Section 4, demonstrates the modelling approach for the case of a reinforced beam. It includes the calibration of the proposed distribution functions for the spatial variation of corrosion. In current guidelines the impact of corrosion is modelled via empiri-

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Nomenclature

A_{ave}	average cross-section reduction of rebar (mm^2)
A_{pit}	cross-section reduction of rebar due to localised corrosion calculated by deducting the overlapping area of uniform corrosion (mm^2)
$A^{(i)}_{res(mod)}$	modified total residual cross-section of two rebars after uniform and localised corrosion (mm^2)
A_{uni}	cross-section reduction of rebar due to uniform corrosion (mm^2)
A_0	initial cross-section of rebar (mm^2)
b	beam section width (mm)
b_p	pit width (mm)
d	effective beam height (mm)
d_0	original rebar diameter (mm)
f_c	concrete compressive strength (MPa)
f_y	steel yield strength (MPa)
f_y^D	residual steel yield strength (MPa)
$G_M^{(i)}$	bending limit state function (MPa)
k	shape parameter of gamma distribution (–)
l	length of beam span (mm)
$l^{(i)}$	location of ith pit along rebar (mm)
l_p	distance between pits in two adjacent rebars (mm)
l'_p	distance between pits in rebar (mm)
$l'^{(i)}_p$	distance between $(i-1)$th and ith pit in same rebar (mm)
l_r	distance between two adjacent tensile rebars (mm)
$M_R^{(i)}$	flexural strength at the location of the ith pit (MPa)
$M_S^{(i)}$	bending moment at the location of the ith pit (MPa)
p_{ave}	average corrosion depth (mm)
P_f	probability of failure (–)
p_{max}	localised corrosion depth (mm)
Q_w	self-weight (kN/m)
Q_L	live load (kN/m)
R_p	pitting factor (–)
x_e	random variable of exponential distribution (–)
x_g	random variable of gamma distribution (–)
α_y	empirical coefficient (–)
β	interference factor (–)
θ	scale parameter of gamma distribution (–)
$\Gamma(k)$	gamma function (–)
λ	rate parameter of exponential distribution (–)

cal relations for strength and ductility of the corroded reinforcement bars [16,17]. Localised corrosion is usually treated as uniform corrosion with larger impact coefficients [18]. In Section 4.5 the results of the proposed modelling approach are compared to two empirically based modelling approaches. At this stage, it is emphasised that the case study presented in Section 4 is merely a demonstration of the proposed approach. For more general conclusions additional cases would be required.

2. Modelling the residual capacity of a corroded reinforced concrete element

2.1. Describing the residual geometry of a reinforcement bar

The surface of a corroded bar induced by chloride might be very uneven. The most detailed description of the residual geometry would be a full 3D description of the corroded rebars. In this case, the geometry of the rebars is described as continuously varying cross-sectional shapes along the rebar. Describing the residual geometry of corroded bars in this way is impractical for at least two reasons: detailed information on the full 3D geometry of rebars is typically not available, and using such data to predict the load carrying capacity of the structure is not straightforward. It would require the use of extremely detailed nonlinear finite elements analyses, describing the detailed geometry of pits and ribs along the rebars [19].

A much simpler way of describing the residual geometry is assuming uniform corrosion. The advantage of this simplification is that standard design procedures could be applied, taking into account the reduced cross-section. If it is assumed that the level of uniform corrosion is equal to the average corrosion, i.e. the volume of the corroded bar is the same, it is clear that additional measures have to be taken to keep the design procedure conservative. An additional safety factor might be applied for the uniform cross-section reduction to take into account the actual variability of the cross-sectional area along the bar and to represent the smallest rebar cross section. Besides additional coefficients for the mechanical properties of the bar should be applied to account for the strength and ductility reduction of the bar which are results of local stress localisations [16,17,20].

Stewart & Al-Harthy [7] use a refined geometric description of corrosion in which a RC beam is discretised *a priori* into a series of small elements. The maximum pit depths are generated for each element. With such a model the arbitrary selection of a uniform cross-section reduction is avoided. However, this modelling seems to be less suitable for the focus of the current paper: modelling the effect of interference between corrosion pits, because this model does not explicitly represent the location of pits.

In this paper we used a different geometric description which allows quantification of the interference of localised corrosion. Fig. 2.1 shows the original cross-section (A_0), the cross-section reduction due to uniform corrosion (A_{uni}), and the cross-section reduction due to localised corrosion (A_{pit}).

When inspecting naturally corroded rebars it becomes apparent that the distinction between localised and uniform corrosion is not clearly visible, and the explicit modelling of both requires some simplifying assumption. This is exemplified in Fig. 2.2. If the uniform corrosion is assumed to be equal to the minimum cross-section loss the number of the pits is high. If the assumed uniform corrosion is increased the number of pits will gradually decrease. This paper will adopt this way of modelling the residual geometry of rebars that consists of a uniform corrosion and a discrete number of localised cross-section reductions.

2.2. Interference between pits on adjacent rebars

Localised corrosion affects the residual load carrying capacity of RC elements. It is shown that the cross-section reduction varies along the tensile rebars and that the cross-section reduction differs between rebars [1,22,23]. The disparities of localised cross-section reduction between rebars may result in interference between the pits (see Fig. 2.3).

The interference of pits in a beam with multiple rebars can be numerically assessed by performing a detailed 3D non-linear finite element analyses. Kioumarsi et al. [13–15] selected an idealized case to quantify the possible interference of localised corrosion on adjacent rebars in an RC beam subjected to bending. In the idealized case two adjacent rebars were considered with one corrosion pit each within the maximum bending zone. The two corrosion pits were equal in size. In a series of nonlinear finite element models the combined influence of two variables on the M_{ult}