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A comparative study of models for shear strength of reinforced concrete deep beams

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ABSTRACT

Since the 1960s, researchers have proposed different empirical formulas and analytical models for the shear strength of deep reinforced concrete beams. Some of these approaches have shown adequate accuracy when applied to small sets of beam tests, while their ability to predict the effect of a large range of test variables remains unknown. This paper presents a summary of models for deep beams from 73 publications, and focuses on a detailed evaluation of ten more recent models by using a database of 574 deep beam tests. It is found that a semi-empirical strut-and-tie model (STM) and a two-parameter kinematic theory (2PKT) for deep beams produce the least scattered predictions. The former model produced an average shear strength experimental-to-predicted ratio $V_{\mathrm{exp}}/V_{\mathrm{pred}}$ of 1.00 with a coefficient of variation (COV) of 19.8%, while the latter resulted in an average of 1.08 with a COV of 15.4%. The two models are also compared by plotting the $V_{\mathrm{exp}}/V_{\mathrm{pred}}$ ratios against different tests variables, and by performing parametric studies with individual test series. It is shown that the semi-empirical STM exhibits certain bias with respect to the shear-span-to-depth ratio, while the 2PKT produces uniform results across the entire range of experimental data. It is also noted that the semi-empirical STM requires somewhat less computational effort than the 2PKT approach.

1. Introduction

The shear behaviour of deep reinforced concrete beams has been a subject of intensive experimental studies since the 1950s. It has long been recognized that, due to their small shear-span-to-depth ratios ($a/d \leqslant$ approx. 2.5), deep beams can carry significantly larger shear forces than slender beams. For this reason, deep beams are often used as transfer girders in buildings, cap beams in bridge bents, pile caps in foundations, and other heavily loaded structural members. From a modelling point of view, deep beams do not obey the classical plane-sections-remain-plane hypothesis, and therefore require different models than slender beams. Since the 1960s, researchers have proposed various empirical formulas and analytical models for evaluating the shear strength of deep beams [1–11]. The most commonly used among these approaches is the strut-and-tie approach which is based on the main characteristic of deep beams, that is the direct transfer of forces from the loads to the supports by means of compressive stresses (strut action or arch action) [12]. This approach represents a simple and powerful tool for the design of structures, which typically provides conservative strength predictions. Other approaches have also shown adequate accuracy in predicting shear strengths when compared with small sets of experimental data [13], while their ability to capture the effects of a large range of test variables remains unknown. It is therefore the purpose of this paper to provide a more comprehensive evaluation and comparison of published models for shear strength of deep beams. The models will be compared by using a database of 574 tests of beams with $a/d \leqslant 3.0$. The comparisons will be performed in terms of main physical assumptions, statistical performance, and the ability of the models to capture the effect of different experimental variables on the shear strength of deep beams.

2. Models for shear strength of RC deep beams

As part of this study, 73 papers on models for shear strength of deep beams published between 1987 and 2014 have been reviewed in detail. These models are applicable to beams subjected to single curvature bending under the action of point loads (typically one or two loads). Taking into account their main features, the models are divided into the following six categories: artificial intelligence models, numerical models (i.e. finite element models, FEM, and discrete element models, DEM), strut-and-tie models,

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Nomenclature

Symbols

a: M/V	length of shear span measured from the centre of the support to the centre of the loading plate
a_g	maximum specified size of course aggregate
b	cross section width
d	member effective depth
f_c	concrete cylinder strength at date of testing
f_y	yield strength of flexural tension reinforcement
f_{yv}	yield strength of stirrups
h	member total depth
l_{b1}	longitudinal length of loading plate
l_{b2}	longitudinal length of support plate
L_n	clear span of beam
P	applied concentrated load
T	tension force in bottom reinforcement
V	shear force
V_c	shear resisted by diagonal strut
V_{exp}	experimentally obtained shear strength
V_{pred}	predicted shear strength
V_{CLZ}	shear resisted by the CLZ
V_{ci}	shear resisted by aggregate interlock
V_d	shear resisted by dowel action
V_s	shear resisted by stirrups
V_{2PKT}	shear strength of deep beams acc. to 2PKT approach
V_{sect}	sectional shear strength of slender beams acc. to AASHTO code
V_w	shear resistance provided by web reinforcement
θ	angle of diagonal strut
Δ_c	transverse displacement capacity of critical loading zone
Δ_t	deflection due to elongation of bottom longitudinal reinforcement
$\varepsilon_{t,\text{avg}}$	average strain along bottom longitudinal reinforcement
ρ_h	ratio of longitudinal web reinforcement
$\rho_l = 100A_s/(bd)$	ratio of longitudinal reinforcement on flexural tension side of section
ρ_v	ratio of transverse reinforcement

upper-bound plasticity models, shear panel models, and other mechanical models. As can be seen from Fig. 1, the majority of the 73 publications focus on strut-and-tie models, followed by numerical models, and artificial intelligence models. The smallest category is “other mechanical models” which includes a model proposed by Zararis [6] and a two-parameter kinematic theory (2PKT) proposed by Mihaylov et al. [11].

Out of all reviewed approaches, this study focuses on ten more recent models [2–11] from four categories, excluding numerical approaches and artificial intelligence models. These models were adopted from a previous study by Senturk and Higgins [13] except for two models [10,11] which were published after their study. The selected models are listed in Table 1 which also provides a summary of their main characteristics. Numerical models are excluded from the discussion because they are not developed specifically for deep beams and are significantly more complex than the rest of the approaches. Since one of the goals of this paper is to compare different physical assumptions for the modelling of deep beams, artificial intelligence models are also excluded from the discussion.

As can be seen from Table 1, the selected models are classified as either analytical or semi-empirical. By semi-empirical it is meant that the theoretical model includes factors, which are derived by fitting shear strengths obtained from deep beam tests. The physical assumptions and the range of test variables used in the development of the models impose limits on their applicability, as listed in the last column of Table 1. These limits, as reported in the original papers describing the models, are typically defined in terms of shear-span-to-effective-depth ratios (a/d) or shear-span-to-depth ratios (a/h). For example, limits $a/h \leqslant 2.0$ and $a/h \geqslant 0.23$ are meant to separate deep beams from slender beams and column-like members, respectively. The 2PKT [11] takes a different approach: the model has been developed to apply to members with short shear spans where the shear strength predicted by this model, V_{2PKT}, will exceed the shear strength predicted by sectional design procedures intended for longer spans, V_{sect} [14]. The following subsections provide a brief description of the ten selected models in their respective categories.

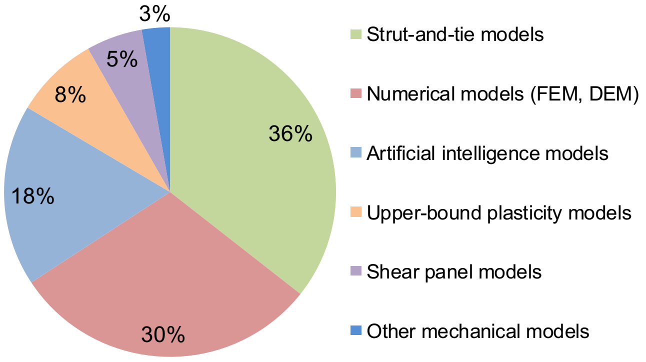

Fig. 1. Models for shear capacity of RC deep beams published between 1987 and 2014.

2.1. Strut-and-tie models

Strut-and-tie modelling is the most commonly used approach for deep beams as demonstrated by the fact that it has been part of design codes since 1984 [15–17]. There are six strut-and-tie models included in this study, three of which semi-empirical and three analytical, see Table 1. In the general case, strut-and-tie models for deep beams include three mechanisms of shear resistance: a direct diagonal strut between the load and the support, a truss mechanism involving the vertical web reinforcement, and a truss mechanism involving the horizontal web reinforcement, see Fig. 2. The struts and ties join in nodal zones in the vicinity of the loading and support points. Hwang et al. [4] assumed that the proportion of shear carried by each of the three mechanisms can be determined based on the angle of the diagonal strut θ. Prior to yielding of the web reinforcement, the proportion of the shear carried by the vertical web reinforcement decreases with increasing θ, and that carried by the horizontal web reinforcement increases with θ. The proportion of shear resisted by the diagonal strut increases up to a strut angle of 45° and decreases for larger angles. Following the yielding of the web reinforcement, the shear increase is carried entirely by the diagonal strut up to the failure of the beam. The failure is assumed to occur due to crushing of the concrete in the vicinity of the nodal zones.