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# Strength of reinforced concrete footings without transverse reinforcement according to limit analysis

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ABSTRACT

Isolated footings are reinforced concrete elements whose flexural and punching shear strengths are usually governing for their design. In this work, both failure modes and their interaction are investigated by means of the kinematical theorem of limit analysis. Previous works in this domain have traditionally considered failure mechanisms based on a vertical penetration of a punching cone. In this work, two enhanced failure mechanisms are investigated considering not only a vertical penetration of the punching cone, but also a rotation of the outer part of the footing, allowing to consider the role of both bottom and top reinforcements on the failure load. A rigid-plastic behavior with a Mohr–Coulomb yield criterion is considered for the concrete and a uniaxial rigid-plastic behavior is assumed for the reinforcement bars. The analysis shows that a smooth transition between flexural and punching shear failure occurs, corresponding to a flexural-shear regime. With respect to the punching shear failure regime, it is shown that the top reinforcement might play an important role (a fact usually neglected by previous investigations). Simplified formulations, allowing easy calculation of the load carrying capacity of footings, are derived and compared to the solutions according to limit analysis. Both theoretical and approximated solutions are finally compared with experimental results, showing consistent agreement.

## 1. Introduction

Concrete footings are commonly used as foundations for buildings and bridges. Although the load carrying capacity of footings subjected to a concentrated loading originated from a column has been the object of different research works [e.g. 1–18], there is still not yet a consensus on a consistent method with physical basis for its design. In this paper, a rational approach is presented on the basis of the kinematical theorem of limit analysis, providing an upper bound solution for the load carrying capacity of these members. The approach may be applied to footings subjected to a distributed soil reaction (as the case of footings with a uniform soil reaction, see Fig. 1(a)) or to footings with concentrated reactions (as the case of pile caps, see Fig. 1(b)).

One of the first applications of limit analysis to reinforced concrete members subjected to in-plane shear was proposed by Drucker [19], who developed both a lower and an upper bound solution for a beam without shear reinforcement (refer to Fig. 2 (a) and (b)). Drucker [19] also showed that the proposed upper and lower bound solutions provided the same failure load and thus corresponded to the exact solution according to limit analysis. According to Drucker [19], failure in shear occurs by crushing of the inclined compression strut (with or without yielding of longitudinal reinforcement). This has been observed to be consistent with experimental evidences only for beams with low slenderness (see Fig. 2(c), for beam B1 of Leonhardt and Walther [20]). For larger slenderness (Fig. 2(d), beam B6 of Leonhardt and Walther [20]), failure occurs instead by an unstable propagation of a critical shear crack developing through the compression strut. In these latter cases, the strength is no longer controlled by the concrete crushing and strain localization occurs. Thus, size effect and other phenomena govern [21,22] and the application of limit analysis is in principle unsuitable for these cases. Analogously to the behavior observed in beams, the strength of slender two-way slabs without shear reinforcement might be governed by the development of a critical shear crack, thus being in the range where limit analysis is not applicable [23]. On the contrary, footings and compact slabs failing in punching can be considered to be similar to beams with low shear slenderness failing by crushing of concrete struts, thus corresponding to the range of cases where limit analysis may be applied.

Limit analysis has already been applied in several cases focusing on the flexural and shear capacity of plain and reinforced concrete

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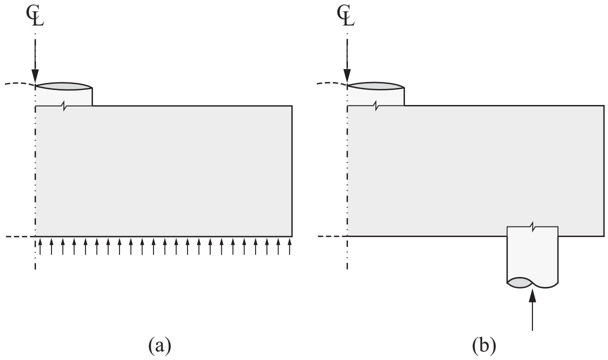

**Fig. 1.** Schematically representation of (a) footing with uniform reaction and (b) pile caps with concentrated reactions.

elements as joints, beams and slabs [e.g. 19,24–46]. With respect to punching shear in slabs, Braestrup et al. [28], Nielsen et al. [31] and Braestrup [32], presented a first theoretical solution based on the kinematical theorem, considering the concrete as a rigid-plastic material with a modified Coulomb yield criterion. The adopted failure mechanism consisted on a vertical shift of the outer slab portion, see Fig. 3. Later, Jiang and Shen [37], Bortolotti [39], Kuang [40] and Salim and Sebastian [42] also applied the upper bound theorem, adopting the same mechanism proposed by Braestrup et al. [28], but with some modifications, namely, in the adopted failure criterion for the concrete.

A drawback of the above mentioned works, based on limit analysis to punching shear in slabs, is that the adopted failure mechanism only considers a vertical displacement along the failure surface, therefore neglecting the possibility of rotations leading to the activation of both bottom flexural and top reinforcement (and thus allowing only the analysis of punching regimes and not flexural or combined flexural-shear regimes). Moreover, all the above mentioned works deal mostly with punching shear strength of general slabs, where the application of this theory becomes potentially questionable (influence of size effect and other phenomena [22]).

In the present work, a theoretical solution for the load carrying capacity of axisymmetric isolated footings with low slenderness is presented. Two different failure mechanisms were selected as potentially governing. Both failure mechanisms consider that two footing portions are separated by a failure surface, which is assumed to be rotationally symmetric. The inner portion is considered to be rigid, while the outer portion deforms due to tangential moments according to a conical shape. Contrary to previous works, the mechanisms considered in the present paper lead to the consideration not only of the internal energy dissipated along the failure surface, but also of the internal energy dissipated in the bottom and top reinforcement, as well as in the concrete compression zone due to tangential bending. The governing failure mechanism is obtained in each case by minimization of the failure load accounting for the fact that both failure mechanisms provide an upper bound solution of the actual failure load.

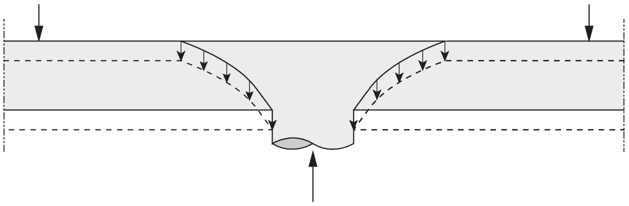

**Fig. 3.** Kinematically admissible failure mechanism proposed by Braestrup et al. [28] and Braestrup [32].

On that basis, simplified solutions are also proposed, consistent with the upper bound solutions developed. Finally, both approximated and optimized solutions are compared with available experimental tests results, showing the consistency and accuracy of the approach.

## 2. Kinematical theorem of limit analysis applied to isolated reinforced concrete footings

In limit analysis, materials are assumed to behave in a perfectly plastic manner [45]. The application of the limit analysis is based on limit state theorems, and, in this paper, the kinematical theorem is used, providing an upper bound of the load carrying capacity. Global equilibrium is investigated stating that the rate of internal energy dissipated has to be balanced by the rate of external work for a licit (kinematically admissible) mechanism.

In this work, a rigid-plastic compressive behavior of concrete with a Mohr–Coulomb yield criterion is assumed, see Fig. 4 (a) and (b). Also the normality condition (strain rate vector normal to the yield locus) is respected. Due to the brittle behavior of concrete in tension, tensile strength is neglected (introduced as a tension cut-off in the plasticity surface). In order to take into account the brittleness of concrete in compression as well as the influence of transverse strains on concrete strength, a plastic compressive strength $f_{cp}$ is considered, which is given by [44]:

$$f_{cp} = f_c \cdot \eta_\varepsilon \cdot \eta_{fc} \tag{1}$$

where $f_c$ refers to the cylinder concrete compressive strength, $\eta_\varepsilon$ and $\eta_{fc}$ represent the reduction factors accounting, respectively, for the presence of transverse strains and for the brittleness of high-strength concrete. Although different approaches have already

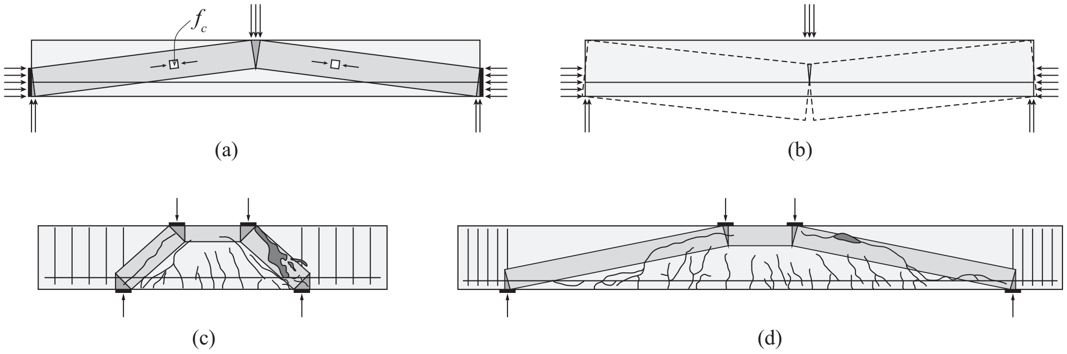

**Fig. 2.** (a) Stress field and (b) kinematically admissible failure mechanism proposed by Drucker [19] for simply supported beams without transverse reinforcement subjected to a single load: cracking pattern and location of theoretical strut of (c) beam B1 and (d) beam B6 by Leonhardt and Walther [20].