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Dynamic response of a multi-layered FRP cylindrical shell under unsteady loading conditions

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ABSTRACT

Taking into account the composite material damping and elastic behavior, the dynamic response of thin-walled FRP pipelines made of symmetric and balanced laminate, under moving singular pressure shock is derived. The methodology is based on double integral transforms and generalized functions' properties. An analytical inversion of the derived Laplace transform is achieved and implementation of the solution on a long multi-layered E-Glass/Epoxy FRP pipeline under fluid-hammer conditions is discussed.

1. Introduction

In last decades, the use of FRP materials in pipeline applications has been gradually increasing throughout the world due to their excellent mechanical and chemical properties. The propagation of pressure waves in liquid-filled pipes is correlated to industrial problems of water hammer and other types of oscillating pressure phenomena [1]. When a pressure shock is traveling along a pipe, flexural waves are generated on its wall. Since both flow characteristics and anisotropic material properties of the laminated wall of FRP pipes are controlling the dynamic displacements, derivation of an accurate model for dynamic response simulation should take into account both elastic and damping parameters [2]. Numerical studies regarding dynamic flow through cylindrical vessels have been reported for pulsating flow in rigid vessels e.g. [3] or fluid-pipe interaction and wave propagation in isotropic pipes e.g. [4]. Apart from the above researches, recent studies have analyzed successfully the dynamic response of filament wound pipes e.g. [5,6], however, they neglect the damping characteristics of the FRP material and contain analyses mostly for harmonic pressure oscillations. Even though some experimental procedures e.g. [7] have been used for strain measurements on CFRP tubes under impulsive loading, according to the author's knowledge there is a lack of models for time-depended radial displacements' prediction for FRP pipelines under transient loading conditions, taking into account both damping and elastic material properties.

In the present work, an analytic solution for radial displacements motion simulation of long multi-layered filament wound pipelines under moving pressure shock of Dirac Delta type $p(x,t) = P_o \cdot \delta(x - a \cdot t)$ is derived. The proposed solution takes into account the FRP material's layout as well as its elastic and damping characteristics. By using Fourier sine and Laplace integral transforms and generalized functions' properties, the derived partial differential equation (PDE) of the dynamic radial displacement $w(x,t)$ is transformed to an algebraic one containing the integral transformation of the variable. The achieved analytical inversion of the Laplace transform and numerical inversion of Fourier transform yields the final solution. The proposed solution has the following advantages: (a) It takes into account the FRP material damping properties as well as the translational inertia and elastic resistance effects; (b) It is expressed in terms of simple and easily calculated integrals, without infinite series or complex variables. Since the study of fluid–structure interaction phenomena is beyond the targets of this research, the derived solution can be used only as an approximation of FRP pipelines' response in fluid hammer conditions.

2. Dynamic radial displacement model for the wall of an FRP cylindrical shell made by a symmetric and balanced laminate

2.1. Free vibrations (no damping)

The model of the laminated axisymmetric thin-walled shell shown in Fig. 1 is used to derive the dynamic equation of motion. Let's denote by w, u, υ the displacement components from the

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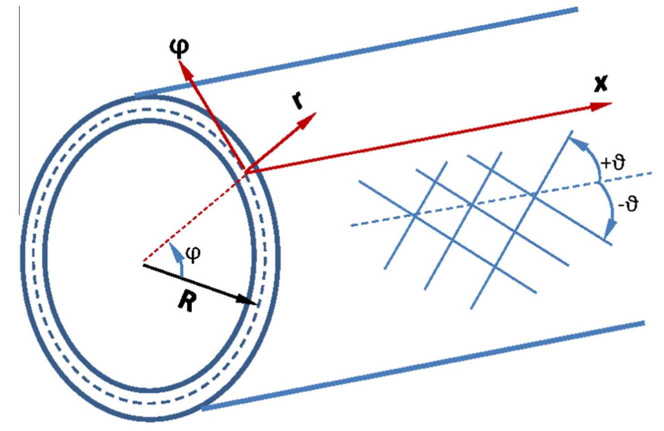

Fig. 1. Coordinate system for a FRP cylindrical shell made by a symmetric and balanced laminate.

middle surface in the r, x, φ direction respectively. The following matrix equation provides the relationship between the stress resultants applied to the wall's laminate, and the middle surface strains and curvatures:

$$\begin{Bmatrix} N_x \\ N_\varphi \\ N_{x\varphi} \\ M_x \\ M_\varphi \\ M_{x\varphi} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_\varphi^o \\ \gamma_{x\varphi}^o \\ k_x^o \\ k_\varphi^o \\ k_{x\varphi}^o \end{Bmatrix} \tag{1}$$

In the above equation the parameters A_{ij}, B_{ij}, D_{ij} (i,j = 1, 2, 6) are given by the following relations:

$$A_{ij} = \sum_{k=1}^{N} \overline{Q}_{ijk}(z_k - z_{k-1}) \tag{2}$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{N} \overline{Q}_{ijk}(z_k^2 - z_{k-1}^2) \tag{3}$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} \overline{Q}_{ijk}(z_k^3 - z_{k-1}^3) \tag{4}$$

For each layer k (Fig. 2) the parameters $\overline{Q}_{ijk}$ are given by the following expressions:

$$\overline{Q}_{11k} = Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4 \tag{5}$$

$$\overline{Q}_{12k} = (Q_{11} + Q_{12} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4) \tag{6}$$

$$\overline{Q}_{22k} = Q_{11}n^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}m^4 \tag{7}$$

$$\overline{Q}_{16k} = (Q_{11} - Q_{12} - 2Q_{66})m^3n + (Q_{12} - Q_{22} + 2Q_{66})mn^3 \tag{8}$$

$$\overline{Q}_{26k} = (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n \tag{9}$$

$$\overline{Q}_{66k} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})m^2n^2 + Q_{66}(m^4 + n^4) \tag{10}$$

$$m = \cos\vartheta, \qquad n = \sin\vartheta \tag{11}$$

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}},$$
$$Q_{66} = G_{12} \tag{12}$$

The material constants E_{11}, E_{22} are the modulus of elasticity, ν_{12}, ν_{21} are the Poisson's ratios, and G_{12} is the shear modulus in the principal directions 1–2.

For the case of symmetric and balanced laminates the following simplifications e.g. [8] are valid:

$$B_{ij} = 0, \quad A_{16} = 0, \quad A_{26} = 0 \tag{13}$$

To derive the Green's function for a multi-layered FRP pipeline under moving singular pressure shock, the following motion equation [5] will be solved:

$$L_{13}u + L_{23}\upsilon + L_{33}w = -p \tag{14}$$

where

$$L_{13} = \frac{A_{12}}{R}\frac{\partial}{\partial x} + \frac{A_{26}}{R_2}\frac{\partial}{\partial \varphi} - B_{11}\frac{\partial^3}{\partial x^3} - \frac{3B_{16}}{R}\frac{\partial^3}{\partial x^2 \partial \varphi} - \frac{B_{12} + 2B_{66}}{R^2}\frac{\partial^3}{\partial x \partial \varphi^2}$$
$$- \frac{B_{26}}{R^3}\frac{\partial^3}{\partial \varphi^3} \tag{15}$$

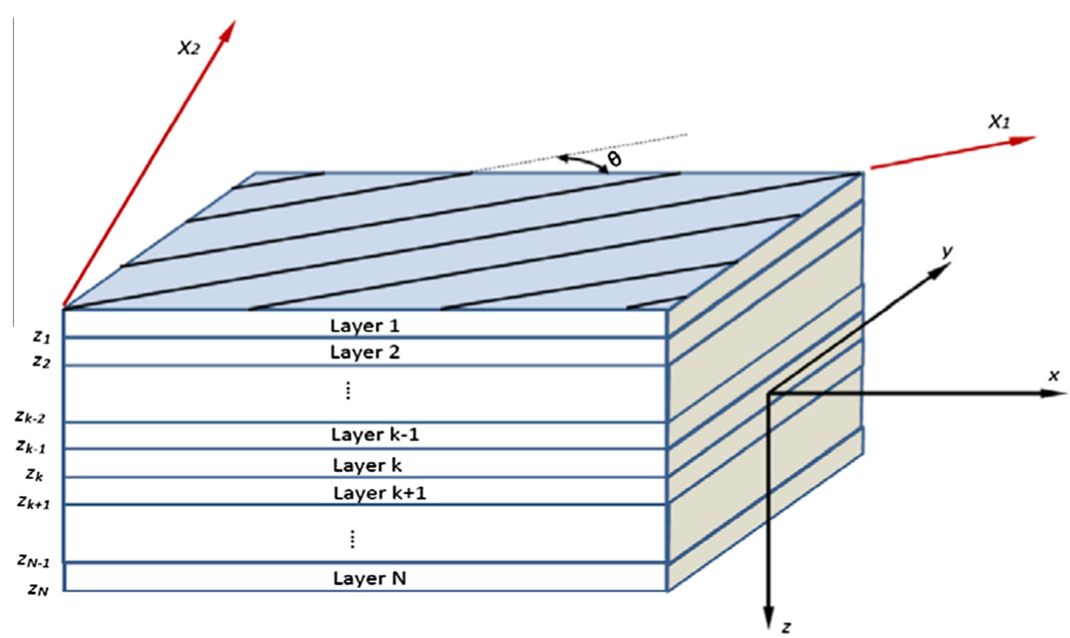

Fig. 2. Stacking sequence of the laminated material.