Engineering Structures 111 (2016) 394–410

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Toward combined local-average stress field modeling of reinforced concrete

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ARTICLE INFO

Article history:
Received 6 February 2014
Revised 20 November 2015
Accepted 23 November 2015
Available online 7 January 2016

Keywords:
Nonlinear finite element
Reinforced concrete
Smeared crack
Local stress field

ABSTRACT

Local mechanisms in RC domain are highly related to cracking initiation/propagation and reinforcement arrangement. These mechanisms and their significant role in the determination of average constitutive models are investigated in this paper. Developing a FORTRAN-based nonlinear finite element program, effects of changing local mechanisms' state during loading procedure are introduced into average tension-softening/stiffening behavior of concrete and average yield strength of steel bar. Adopting a "post-process" procedure, a combined local-average finite element modeling of RC members is conceivable. Accordingly, adequate displacement values are detected at the crack plane and accurate displacement-based constitutive models are applied instead of simplified strain-based ones.

1. Introduction

Due to the complex phenomena involved in nonlinear response of RC members, there always exists a world-wide interest in simplification of the effective participating mechanisms in developing numerical simulations and constitutive models. This interest applicably concerned by the University of Toronto since 1985 [1] by organizing an international competition for analytical evaluation of four RC panels [2].

Among several stated approaches, "smeared crack" is known as a simple and reliable method for evaluation of RC members' behavior with a good estimation on strength and crack pattern. In this approach, each expected phenomenon in post-crack behavior of RC members is interpreted by an average constitutive model which is developed for "*between-cracks domain*". By this fact, the more reliable the crack domain is detected, the more validity will be introduced into the approach. Therefore, it is noteworthy to find a method in which the cracking state is detected closer to the real state.

Based on the smeared crack concept, two main finite element-based researches have been widely conducted during last three decades. "Rotating smeared crack", adopted by Vecchio and Collins [3–5], has been extended as MCFT and applied in Vector program, and "Fixed smeared crack" investigated by Maekawa et al. [6,7], is implemented in WCOMD program. The main difference of these two approaches is in ignoring or considering the shear transfer mechanisms at the crack plane.

In common finite element application of the smeared crack approach, the average behavior of a specific zone is assigned to all nodes and elements located in this zone. The mentioned zone is determined by some special theories which differ by an author to another, but the common fact between existing researches is that this zone will remain constant during the analytical procedure. The fundamental phenomenon considered in the definition of this zone is the transfer of bond stress between concrete and steel bar, and this is directly affected by crack initiation and propagation. Since the cracking state continuously differs from the first load step to the end, the definition of a constant zone will not be completely reliable.

To overcome this shortcoming, far from introducing conservative assumptions into the smeared crack approach by the mentioned researches, two sets of studies have been conducted on proper detection of crack initiation and propagation. In the first set, researchers like Broms [8–11], Goto [12], Bažant and Oh [13] and Sato and Fujii [14] represented specific criteria for the development and continuance of cracks; and in the second set, introducing local behaviors into average concepts was considered. Sato and Naganuma [15], Soltani [16–20] and Cerioni [21] modified average stress field approaches by introducing the local phenomena's effects into the average constitutive models; while Avila [22], Oliver [23,24] and Pimentel [25] used additional techniques for local modeling of cracks, e.g. embedded and extended crack approaches.

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http://dx.doi.org/10.1016/j.engstruct.2015.11.035
0141-0296/

Soltani and Maekawa [18] investigated the post-crack characteristics of RC elements in the *between-cracks domain*, considering all local mechanisms and their effects on the average behavior of this domain. By their research, a slip–strain relation was presented for embedded steel bar in concrete, which results in a more exact evaluation of crack spacing and displacement domain at the crack surface.

Retaining the whole idea introduced by Soltani et al. [16–20], authors represent a nonlinear finite element program, ESMARC (enhanced smeared crack analysis of RC members), which is used to analyze the RC members' behavior based on the smeared crack approach. In order to introduce the local stress field into the average one, a "Post-Processing" concept has been added to the main finite element body of the program to evaluate the local mechanisms by meso-scale calculations.

This localized post-processing enriches the adopted common smeared crack by considering local and average characteristics simultaneously. It concentrates on the probable local mechanisms in the between-cracks length, therefore it can easily evaluate the effect of any parameter, e.g. rebar diameter, reinforcement ratio and direction, crack direction, and embedment length on the cracking procedure and the average behaviors of steel bars and concrete. The proposed post-processing, also introduces more exact values for displacement domain, i.e. crack spacing, opening and sliding. Accordingly, the displacement-based constitutive models are introduced with no simplifications nor conservative assumptions in the main finite element body of the program.

It is remarkable that in comparison with existing analytical approaches for modeling and evaluation of RC members' behavior, the proposed method will reach to a reliable estimation of stiffness, strength, crack pattern and total average behavior, while there will be no significant increase in the computational cost and time. On the other hand, the ability of this approach to detect more exact values for crack spacing is the outstanding feature of this method.

The proficiency of the proposed approach is numerically investigated for several uniform and non-uniform cases, comparing the analytical results with the experimental ones. In addition, the efficiencies of this approach in considering the local features and mechanisms are discussed for some examples.

2. Nonlinear finite element formulation of smeared crack approach

In this study, nonlinear analysis of RC members is numerically investigated by common finite element algorithms [26]. For this purpose, a finite element based program, introduced as 'ESMARC', has been developed by FORTRAN. Isoparametric quadrilateral elements with 4 & 8 nodes have been defined for meshing, each with 4 or 9 integration points.

Finite element analysis procedure of an RC integration point is divided into two main phases, pre- and post-crack phases; and the state of an integration point, in this research, is detected by the principal tensile strain. Therefore, the point will be in pre-crack phase unless its' principal tensile strain exceeds from the cracking strain of concrete. A brief chart is presented in Fig. 1 to show the evaluation progress of an integration point and the states for applying the constitutive models. In addition, Fig. 2 expresses the stress state for a reinforced concrete member in box (I), including the presence and direction of reinforcement sets. It also represents the models adopted to introduce the behavior of concrete and steel bars in post- and pre-crack phases, in the boxes (II) and (III), respectively.

In the pre-crack phase, it is recommended to model the response of an RC member by use of the main concepts adopted in the common smeared crack method. Whereas in the post-crack phase, the behavior of the member is represented by summation of the individual behavior of concrete and steel bars (Eq. (1)). Based on several experimental and numerical researches conducted by Maekawa et al., non-orthogonal 2-way fixed smeared crack has been applied as the main algorithm of modeling the total RC behavior in the post-crack phase [6]. For this case, concrete's behavior is defined by two uni-axial behaviors, one in the direction parallel to the crack plane and the other in its' normal direction. The axial behavior of steel bar, whether tension or compression, is added to the behavior of concrete with the aid of the corresponding models and relations.

To obtain the integration point's stiffness matrix, material stiffness matrix for reinforcing bars and concrete must be derived based on the constitutive models which expresses the point's state. Among several theories applicable for concrete's behavior, Darwin's theory [27] is adopted for interpreting the cracked concrete stiffness matrix in the local coordinate system ($[E_{con}(12)]$) and its' transition into the global coordinate system ($[E_{con}(xy)]$). In the presented equations (Eqs. (2)–(4)), E_1 and E_2 are concrete's tangent or secant modulus in the directions normal and parallel to the crack, respectively; υ_1 and υ_2 are their corresponding poison's value; and G is the shear modulus at the crack plane. $[T]$ is the transition matrix and θ is the angle between the direction normal to the crack and x-axis, as discussed in Fig. 2. It is remarkable that the "x-axis" relates to the horizontal direction in the global coordinate system. Stiffness matrix of the steel bar in the global coordinate system, ($[E_{stl}]$), is also developed by Eqs. (5) and (6), considering the contribution of each reinforcement set (η) individually. Each set is defined by a direction to x-axis (α_η), ratio (ρ_η) and axial stress (σ_η). The equivalent stress matrix for the whole reinforcement sets, $[\tilde{\sigma}_{Sxy}]$, is determined by adopting a summation on the projection of axial behavior of each steel set in x and y direction [19].

$$[E] = [E_{con}] + [E_{stl}] \tag{1}$$

$$[E_{con(xy)}] = [T]^t \cdot [E_{con(12)}] \cdot [T] \tag{2}$$

$$[E_{con(12)}] = \frac{1}{1-(\upsilon_1\upsilon_2)}\begin{bmatrix} E_1 & \sqrt{\upsilon_1\upsilon_2E_1E_2} & 0 \\ \sqrt{\upsilon_1\upsilon_2E_1E_2} & E_2 & 0 \\ 0 & 0 & (1-(\upsilon_1\upsilon_2))\times G \end{bmatrix} \tag{3}$$

$$[T] = \begin{bmatrix} \cos^2(\theta) & \sin^2(\theta) & \sin(\theta)\cdot\cos(\theta) \\ \sin^2(\theta) & \cos^2(\theta) & -\sin(\theta)\cdot\cos(\theta) \\ -2\sin(\theta)\cdot\cos(\theta) & 2\sin(\theta)\cdot\cos(\theta) & \cos(2\theta) \end{bmatrix} \tag{4}$$

$$[\tilde{\sigma}_{Sxy}] = \begin{bmatrix} \sum\limits_{\eta=1,\ldots} \rho_\eta\tilde{\sigma}_{s\eta}\cos^2(\alpha_\eta) & \sum\limits_{\eta=1,\ldots} \rho_\eta\tilde{\sigma}_{s\eta}\sin(\alpha_\eta)\cdot\cos(\alpha_\eta) \\ \sum\limits_{\eta=1,\ldots} \rho_\eta\tilde{\sigma}_{s\eta}\sin(\alpha_\eta)\cdot\cos(\alpha_\eta) & \sum\limits_{\eta=1,\ldots} \rho_\eta\tilde{\sigma}_{s\eta}\sin^2(\alpha_\eta) \end{bmatrix} \tag{5}$$

$$[E_{stl(xy)}] = \begin{bmatrix} \tilde{\sigma}_{Sxy}(1,1)/\varepsilon_x & 0 & 0 \\ 0 & \tilde{\sigma}_{Sxy}(2,2)/\varepsilon_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{6}$$

In combination with the main algorithm of finite element-based smeared crack approach, "Direct Displacement Control" method, presented by Jirasek and Bažant [28], has been adopted as the nonlinear solution method compatible for solving force- and displacement-control problems.

3. Average constitutive models

The proposed approach follows the whole concept of the common smeared crack approach for the pre-crack phase. However, in the post-crack phase, enhanced constitutive models are applied instead of pre-defined models.