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A hybrid damage detection method using dynamic-reduction transformation matrix and modal force error

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ABSTRACT

Condensation methods are commonly used algorithms to fast computation of some low frequencies and corresponding mode shapes of structures by reducing a full rank domain to a restricted one. Hence, these methods are often termed as model reduction techniques. On the other hand, these methods have been used to develop damage detection techniques by several researchers. In this study, a new damage detection method is proposed which uses only one mode shape and its corresponding eigen-value of a structure to conduct damage detection. However, since the number of measured degrees of freedom (DOFs) is most of the time restricted in practice, an iterative hybrid method is developed using dynamic-condensation scheme to carry out damage detection in structures using incomplete modal data. The main characteristic of the proposed method is that measuring the rotational DOFs, which is most of the time very expensive and inaccurate, is not needed. To examine the capability of the proposed method, several examples are studied. In the illustrative example of a cantilever beam, it is assumed that just translational DOFs corresponding to the first mode are available. The capability of the proposed method in damage detection in the cantilever beam and model updating is completely examined in this example. Also a modal assurance criterion is applied to evaluate the robustness of the method in updating the mode shapes. Finally, the ability of the proposed method in damage detection in 2D and 3D frame structures is assessed. Interestingly, the results in most of the cases are exact even with the lack of measured translational DOFs and also demonstrate that the proposed method is completely successful in conducing damage detection in structures.

1. Introduction

Any undesirable defect in structural elements can endanger the integrity and performance of the whole structure. Damages can bring about undesirable eventualities such as catastrophic failure of structures. Therefore, a field of research known as non-destructive damage detection (NDD) has been developed and been mainly used in structural and mechanical engineering applications through the past decades [1]. Although, many experimental techniques such as ultrasonic, radiography, magnetic particle and eddy current have been applied for damage detection in structural components, these methods can be applied to detect damages in a limited part of a structure [2]. That is why these techniques cannot be applied to large structures such as aerospace, bridge and offshore structures alone and applying a global NDD method complying with a local technique would be more viable and effective.

Damage detection can be termed as a procedure in which two different phases of a structure are compared; the initial and damaged states [3]. Therefore, in order to execute damage detection in a structure, some mechanical characteristics of the pristine and the damaged state of the structure are needed. Since the mechanical characteristics of the FE model are often available, most of the time in the literature an undamaged state of a structure is considered as the initial state. That leaves only to find some mechanical characteristics of that damaged structure. One of the easiest methods to identify mechanical characteristics of structures under service is termed as natural excitation techniques (NExT). It is shown that under an ambient-excitation, the cross-correlation function between two different measured responses on a structure has the same analytical form as the free vibration response of the structure [4,5]. This is the basic rule beyond applying ambient vibration data as a significant help with the problem of modal parameter identification. However, the identification of the modal

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parameters such as natural frequencies and mode shapes in large-scale structures under ambient vibration is proved to be a difficult task [6–8].

A model updating method viewed as an inverse procedure is an optimization process aimed at calibrating physical parameters of an analytical model based on experimentally measured data. Some of these physical parameters are the stiffness, damping, and mass properties of structures which can be used to predict damage or evaluate structural integrity. Looking for unknown physical parameters of a structure under service has been paid attention in the literature by several researchers [9–13].

One of the most important mathematical methods applied in structural dynamics is the generalized eigen-value technique which has been widely used in structural vibration analysis, finite element (FE) model updating, damage identification, and so forth. As far as the calculation accuracy of the structural dynamic characteristics is concerned, the number of measured DOFs in FE analysis is needed to be larger in practice. On the other hand, for a large structure, only the first few lower modes usually can be detected applying dynamic tests. Hence, solving the complete eigen-problem is not viable in many cases.

One of the most helpful applied methods to fast computation of the lowest eigen-values and related eigen-vectors of structures is the condensation technique. There are three basic model reduction techniques which are the static or Guyan condensation [14], the improved reduced system (IRS) [15], and the dynamic-condensation methods [16]. The main objectives of these strategies are to eliminate some DOFs of the original FE model (slave DOFs) and retain far less number of DOFs (master DOFs), and solve the eigen-function of the reduced system to approximate the eigen-solutions of the original model.

The static condensation method is one of the oldest and widely used condensation techniques first proposed by Guyan. In this method, the inertia terms of the slave DOFs are neglected, thus it is just accurate at zero frequency. Static condensation method is suitable to be applied to some damage detection procedures which use stiffness matrix of structures. This method is mainly applied when expansion of a condensed stiffness matrix of a structure to global domain is needed. For instance, in a study conducted by Rodrigues et al., an inverse process of static condensation was applied to expand the reconstructed condensed stiffness matrix of damaged structures to the global domain [17]. In a similar study conducted by Escobar et al., the geometric transformation matrix of the static condensation method was used in an iterative damage detection technique [18]. In this paper, the condensed stiffness matrix of a structure is updated through an iterative damage detection procedure.

Since obtaining the stiffness matrix of structures is considered as a difficult task and needs a static test in zero frequency, it seems to be inapplicable for large structures. Moreover, applying the modal data of a structure in an excitation test to calculate stiffness matrix of that structure will increase the errors in damage detection procedures. Hence, applying the dynamic condensation scheme with a damage detection procedure will be more applicable for real-scaled structures. Besides, in an experimental measurement, the number of the measured DOFs is usually restricted practically. Especially for large-scaled structures, the number of measured DOFs is far less than the total number of DOFs of the whole structure. In such situations, mode shape expansion techniques can be applied to expand the experimental mode shapes to the global domain [19]. Similarly, model reduction techniques can be used to condense the FE model to the restricted domain of measured mode shapes [20].

The main superiority of the dynamic condensation method compared with the Guyan and IRS methods is not to introduce any errors in the transformation process within a certain frequency range. Moreover, since a more accurate reduced model is attainable, the damage information from original model is better retained. After studying damage detection techniques and surveying the literature the authors have come to this conclusion that there is a lack of applying the dynamic condensation technique to damage detection in structures.

In the current study, a new method is proposed for conducting damage detection in structures. In this study, damage in an element is simulated as degradation in its stiffness matrix contributing to the stiffness reduction of the whole structure. It is assumed that some elements of a structural mode shape related to the measured master DOFs are available. In the current study, the transformation matrix of the dynamic condensation technique is used in a coupled recursive procedure to update damage indices. Through the procedure of the proposed method the stiffness matrix, mode shapes, and frequencies of the whole structure are updated as well. Several examples in this research are studied. In the first one, an illustrative example of a damaged cantilever beam is studied and the capability of the proposed method in damage detection and model updating is examined thoroughly. In this example, it is assumed that just the translational DOFs corresponded to the first mode are measured. Additionally, in order to evaluate the updated model of the cantilever beam, a modal assurance criterion (MAC) is applied. The obtained MAC data also guarantee the robustness of the proposed method in updating a FE model as well. Moreover, to evaluate the capability of the proposed method in detecting damages in frame structures, two comprehensive examples of 2D and 3D building frames are studied. The results show the merits of the proposed method in indicating damaged and intact elements in frame structures even with the lack of the measured translational DOFs.

2. Theoretical description

2.1. A damage detection method based on the modal force error

One way to confront damage in structures, in the context of structural health monitoring, is to consider it as the degradation of the stiffness matrix of structures. Most of the researches in the literature, do not concern about degradation in mass properties of a damaged structure. In this study, both mentioned assumptions are taken into account as well.

To begin, let's pay attention to the free vibration equation of the FE model of a damaged structure which is presented as follows:

$$(\mathbf{K}_{\mathrm{d}} - \lambda_{\mathrm{dj}}\mathbf{M})\boldsymbol{\phi}_{\mathrm{dj}} = \boldsymbol{0} \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}_{\mathrm{d}}$ represent $n \times n$ analytical mass and damaged stiffness matrices, and λ_{dj} and $\boldsymbol{\phi}_{\mathrm{dj}}$ are the jth modal eigen-value and eigen-vector of the damaged state of that structure, respectively. On the other hand, one can obtain $\mathbf{K}_{\mathrm{d}}$ through extracting matrix $\Delta\mathbf{K}$ from stiffness matrix of the undamaged structure $\mathbf{K}$ as follows:

$$\mathbf{K}_{\mathrm{d}} = \mathbf{K} - \Delta\mathbf{K} \tag{2}$$

Substituting Eq. (2) in Eq. (1) leads to the following equation:

$$(\mathbf{K} - \lambda_{\mathrm{dj}}\mathbf{M})\boldsymbol{\phi}_{\mathrm{dj}} = \Delta\mathbf{K}\boldsymbol{\phi}_{\mathrm{dj}} \tag{3}$$

By taking $(\mathbf{K} - \lambda_{\mathrm{dj}}\mathbf{M})\boldsymbol{\phi}_{\mathrm{dj}} = \boldsymbol{b}_{\mathrm{j}}$ and defining $\boldsymbol{b}_{\mathrm{j}}$ as jth modal force error, Eq. (3) can be summarized as:

$$\Delta\mathbf{K}\boldsymbol{\phi}_{\mathrm{dj}} = \boldsymbol{b}_{\mathrm{j}} \tag{4}$$

Referring to the Continuum Damage Mechanics [21], the damage in ith element can be represented by introducing coefficient α_{i} to the stiffness matrix of the element. This degradation of