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Assessment the stability of masonry walls by the transfer-matrix method

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ABSTRACT

Masonry walls are very sensitive to flexural effects due to low tensile strength which, in turn, greatly influences the load bearing capacity under compression. The main source of flexural effects may results from the eccentric loading at the ends of the wall or from any lateral loading like the wind action, the earth pressure, or the second order effect of the applied actions. Several analytical solutions were proposed in literature to solve the differential equation of the problem, but those solutions were limited to special conditions. In the current contribution, a general formulation for the non-linear stability problem has been formulated numerically based on the transfer-matrix method. Despite the method is out of professional use today and don't possess the potential and flexibility of the finite elements but for the current addressed problem, it is still the most efficient.

A relative form description has been introduced to formulate the stability theory of masonry walls. This description has been used to minimize the dimensions of matrixes in the transfer-matrix method and to produce the equations in a compact form. The algorithms of the method have been derived for general boundary and loading conditions with a user-defined non-linear material model. Algorithms and solution procedures have been explained and implemented into a computer code. The convergence of the iterative solution has been studied with clear definition for the cases at which the stability or material failures occur. The results of the developed solution procedure have been validated by comparing them with the existing solutions and the experimental results. The developed solution procedure provides a powerful tool to solve a wide range of problems related to stability of masonry walls and to check the existing empirical methods.

1. Introduction

The load bearing capacity of a masonry wall is influenced by the eccentricity of load and slenderness ratio, which in turn depends on the geometry, the stiffness of the cross-section, the boundary conditions, and the existence of any lateral load. Masonry walls may have different positions and functions in the structure which might be subjected to a different type of loading and boundary conditions (Fig. 1).

It is more appropriate for practical use and standards to describe the influence of these factors using a **capacity reduction factor** Φ for the compressive strength allowing the actual conditions. Since masonry material has low tensile strength, the wall may crack under certain conditions leading to further complications due to the reduction in the effective cross-section. Masonry members under compression might fail either because of material overstressing for squat members or because of stability failure for slender members. For squat masonry members, the failure takes place if the compressive strain at any cross-section reached the ultimate compressive strain of the material. Nevertheless, for slender masonry elements the failure occurs before reaching the ultimate compressive strain of the material at any cross-section. The former mode of failure is called **material failure** and the latter is called **stability failure**. Both failure modes are going to play a crucial role in the determination of the capacity reduction factor.

2. Stability theory of masonry walls

2.1. Background

A masonry wall of height h and thickness t is considered. The wall is subjected to an eccentric compressive load N with eccentricity e_0 at both top and bottom ends. A strip length of the wall equal to 1 is considered assuming that the load is uniformly distributed over the length of the wall. The wall assumed to be freely rotating on the upper and lower edges (pinned–pinned

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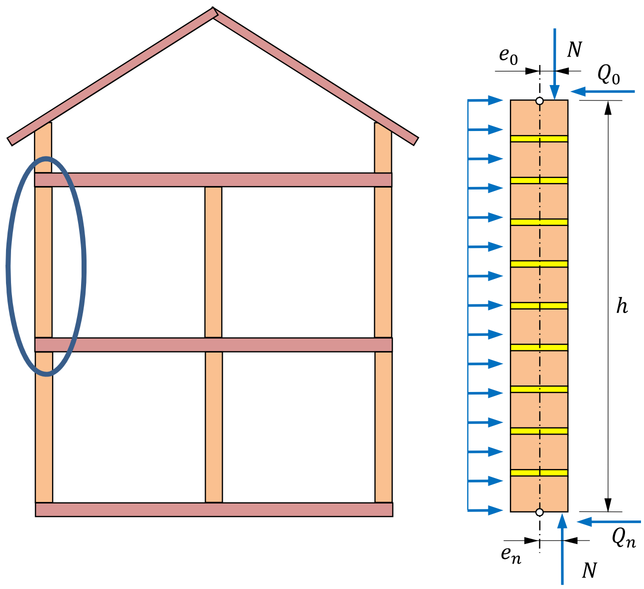

Fig. 1. The bar idealization of masonry wall with the acting loads and boundary conditions.

model) and not supported at the side edges (or it has enough length so that the effect of the side edges boundary conditions can be ignored). Fig. 2 shows the schematic configuration of the wall model which is going to be used to study the buckling of masonry.

The scope of this theory is slender masonry walls $h/t \geqslant 3$. The deformations due to shear have been ignored so that Euler–Bernoulli theory of slender beams can be applied. In Euler–Bernoulli beam theory, the cross-section that is perpendicular to the normal axis of the beam remains plane after bending. That is, no deformations occur in the plane of the cross-section.

2.2. The curvature of deformation

Based on Euler–Bernoulli hypothesis of plane sections the curvature κ of the beam at distance y is given by:

$$\kappa = \frac{1}{\varrho} = -\frac{\frac{d^2e}{dy^2}}{\left(1+\frac{de}{dy}\right)^{\frac{3}{2}}}, \tag{1}$$

where ϱ the radius of curvature of deformation, e is the total eccentricity of the load which is the sum of the eccentricity due to the first order effect e_I and second order effect e_{II}:

$$e = e_I + e_{II}. \tag{2}$$

For small bending deformations, the curvature of deformation can be approximated simply as following:

$$\kappa = -\frac{d^2e}{dy^2}. \tag{3}$$

2.3. Equilibrium and compatibility equations

Masonry is an anisotropic material with low or no tensile strength in comparison to its compression strength. This makes masonry very sensitive to the flexural deformations. It is important to consider the state of damage due to cracking of the cross-section. In Fig. 2 the stress distribution has been plotted on two sections: the first one is uncracked section and the second one is cracked section.

The stress and strain state under flexural deformation in both cross-sections of the wall has been considered. The stress/strain

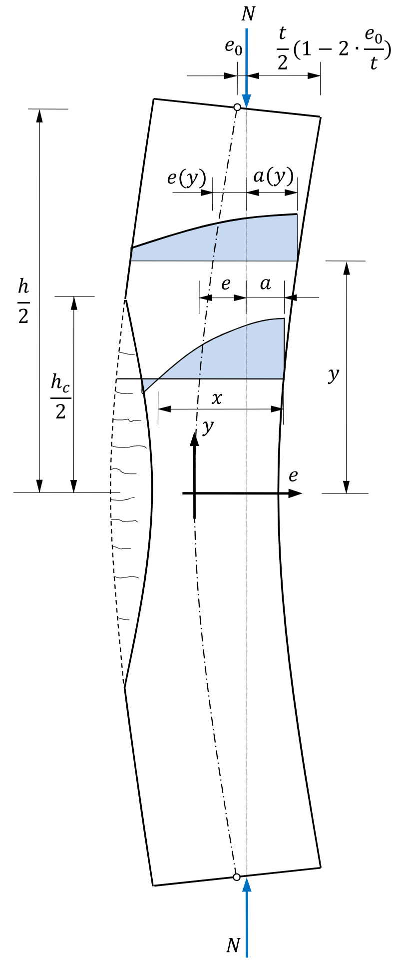

Fig. 2. Schematic drawing of the deformation of a masonry wall due to second order effect.

state at both cross-sections is going to be used to write the equilibrium equations considering the notations defined in Fig. 3.

From the equilibrium equations, the normal force N and the bending moment M can be obtained by integrating the stresses over the area of the cross section:

$$N = \int_A \sigma \cdot dA_w \quad \text{and} \quad M = -\int_A \sigma \cdot z \cdot dA_w, \tag{4}$$

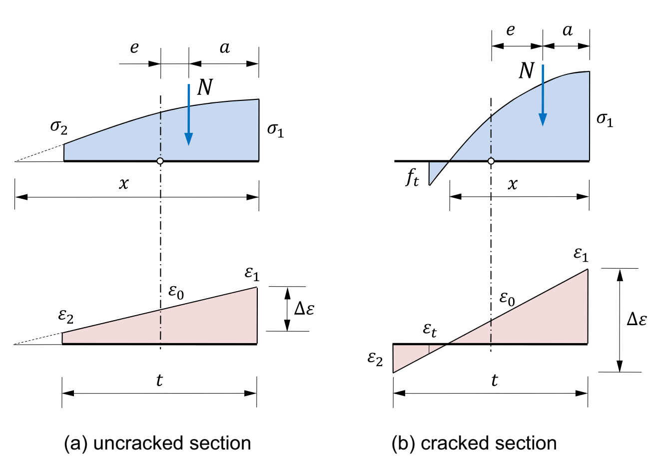

Fig. 3. The stress and strain state in the cross section of masonry wall under flexural deformation.