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Three-dimensional warping in strip-based composite four-bar mechanisms

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ABSTRACT

This work intends to demonstrate the effect of geometrically non-linear cross-sectional analysis of certain composite beam-based four-bar mechanisms in predicting the three-dimensional warping of the cross-section. The only restriction in the present analysis is that the strains within each elastic body remain small (i.e., this work does not deal with materials exhibiting non-linear constitutive laws at the 3-D level). Here, all component bars of the mechanism are made of fiber-reinforced laminates. They could, in general, be pre-twisted and/or possess initial curvature, either by design or by defect. Each component of the mechanism is modeled as a beam based on geometrically non-linear 3-D elasticity theory. The component problems are thus split into 2-D analyses of reference beam cross-sections and non-linear 1-D analyses along the three beam reference curves. The splitting of the three-dimensional beam problem into two- and one-dimensional parts, called dimensional reduction, results in a tremendous savings of computational effort relative to the cost of three-dimensional finite element analysis, the only alternative for realistic beams. The analysis of beam-like structures made of laminated composite materials requires a much more complicated methodology. Hence, the analysis procedure based on Variational Asymptotic Method (VAM), a tool to carry out the dimensional reduction, is used here. The representative cross-sections of all component bars are analyzed using two different approaches: (1) Numerical Model and (2) Analytical Model. Four-bar mechanisms are analyzed using the above two approaches for $\Omega = 20$ rad/s and $\Omega = \pi$ rad/s and observed the same behavior in both cases. The noticeable snap-shots of the deformation shapes of the mechanism about 1000 frames are also reported using commercial software (I-DEAS + NASTRAN + ADAMS). The maximum out-of-plane warping of the cross-section is observed at the mid-span of bar-1, bar-2 and bar-3 are 1.5 mm, 250 mm and 1.0 mm, respectively, for $t = 0{:}5$ s.

1. Introduction

This paper intends to demonstrate the effect of geometrically non-linear cross-sectional analysis of composite beam-based four-bar mechanisms in predicting the 3-D warping of the cross-section. Berdichevsky [1,2] introduced a very useful mathematical method, the Variational-Asymptotic Method (VAM). He applied the method to the beams in [3,4]. Danielson and Hodges [5] developed a kinematical description for these dimensionally reducible structures. In the present study, the kinematics of the problem is formulated using this procedure before applying the VAM to beams. Based on this approach, Cesnik and Hodges [6] solved linear cross-sectional problems for materials having generally anisotropic and inhomogeneous properties. Popescu et al. [7] studied the obliqueness effects in asymptotic cross-sectional analysis. Rajagopal and Hodges [8] proposed a beam theory for the in-plane deformation of an initially curved composite strip. Later it was generalized by Yu and still maintaining this cross-sectional analysis tool, VABS [9].

Popescu and Hodges [10] have introduced geometric nonlinearity at cross-sectional level by discretizing the cross-section of a strip-like composite beam into finite-elements and the trapeze effect, a non-linear effect, is caused by the presence of certain non-linear terms in the strain field because of moderate local rotation [11]. Harursampath et al. [12] studied non-linear cross-sectional problems and developed closed-form analytical solutions for fiber reinforced laminated beams with thin rectangular

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cross-sections. In the present study, we are using these closed-form analytical solutions for the cross-sectional properties and warping of such beams. Giavotto et al. [13] developed a numerical, finite-element based cross-sectional analysis of beams. Chang and Yu [14] modeled composite beams considering spanwise heterogeneity. Borri et al. [15] explained the appropriate use of Hamilton's Weak Principle (HWP) in the developement of consistent and efficient approximations for the determination of the response of mechanical systems. Thuc and Jaehong [16] presented a general geometrically non-linear model for thin-walled composite space beams. Hodges et al. [17] first used the asymptotic methods to develop cross-sectional analysis of anisotropic beams based on the finite-element method (FEM). Cesnik and Hodges [6] solved linear cross-sectional problems for materials having generally anisotropic and inhomogeneous properties based on the approach presented by Danielson and Hodges [5]. Hodges and his co-workers [17,6,18–21,7] introduced many generalizations such as initial twist, curvature and obliqueness effects to develop the code further to a more useful tool. Yu et al. [22] generalized the problems solved by Cesnik and Hodges [6]. Borri and Merlini [23] independently presented a large displacement analysis of beams. Biot [24] and Goodier [25] examined the increase in torsional rigidity due to axial stress. Renton [26] proposed a general beam theory which is applicable to all regular prismatic systems. Hodges [27] examined the effect of tension–torsion coupling terms for a twisted rotating blade.

Bauchau and Kang [28] presented a multi-body formulation for helicopter non-linear dynamic analysis. Bauchau [29] developed two computational schemes, energy preserving and energy decaying schemes for the dynamic analysis of flexible, non-linear multi-body systems. Bauchau and Hodges [30] studied the behavior of non-linear multi-body systems involving elastic members. Bauchau's commendable and comprehensive work on this area is based entirely on numerical techniques, while the current work is partly analytical. An integrated approach for efficient and accurate analysis of composite structures by connecting high-fidelity models of composite beams with a versatile multi-body dynamic environment is introduced by Yu [31]. Pollayi and Harursampath [32] demonstrated the importance of geometrically non-linear cross-sectional analysis of certain composite beam-based four-bar mechanisms in predicting system dynamic characteristics and analyzed two-different ways of arranging the beams in the mechanism, named as width-wise and thickness-wise connections, and simulated using the available commercial software (I-DEAS + NASTRAN + ADAMS) to see the practical applicability by arranging the beams in the mechanism(s) in distinctly different orientations w.r.t. each other.

2. Non-linear dynamic analysis of composite beam systems

A general framework to construct accurate reduced models for composite dimensionally reducible structures such as beam(s) has been formulated based on the two theoretical foundations: (1) Concept of Decomposition of the Rotation Tensor, and (2) VAM. The purpose of the non-linear analysis of the beam cross-section is to determine the elements of the non-linear 1-D stiffness matrix, S^{NL}, and the recovering relations. Recovering relations, for example, provide the relationship between the 3-D strain tensor components, Γ_{ij} $(i,j = 1,2,3)$, and the generalized 1-D strain measures, $\underline{e}^{\star}$. In the present notation, the 1-D constitutive law can be written as:

$$\underline{f}^{\star} = S^{NL}\,\underline{e}^{\star} \tag{1}$$

where $\underline{f}^{\star} = [F_1\, F_2\, F_3\, M_1\, M_2\, M_3]^T$ and $\underline{e}^{\star} = [\gamma_{11}\, 2\gamma_{12}\, 2\gamma_{13}\, \kappa_1\, \kappa_2\, \kappa_3]^T$. The numerical analysis is capable of treating cross-sections of arbitrary geometry and material distributions. Here the arbitrary cross-sections are meshed with finite elements and the FEA is performed to get the non-linear stiffness matrix. Fig. 1(i) shows the sketch of a cantilever beam and its cross-section. For developing a rigorous non-classical non-linear asymptotic theory for beams which could be considered as pretwisted arbitrary assemblies of thin rectangular members. Fig. 1(ii) shows the sketch of an initially twisted and curved strip configuration, coordinate system and cross-section.

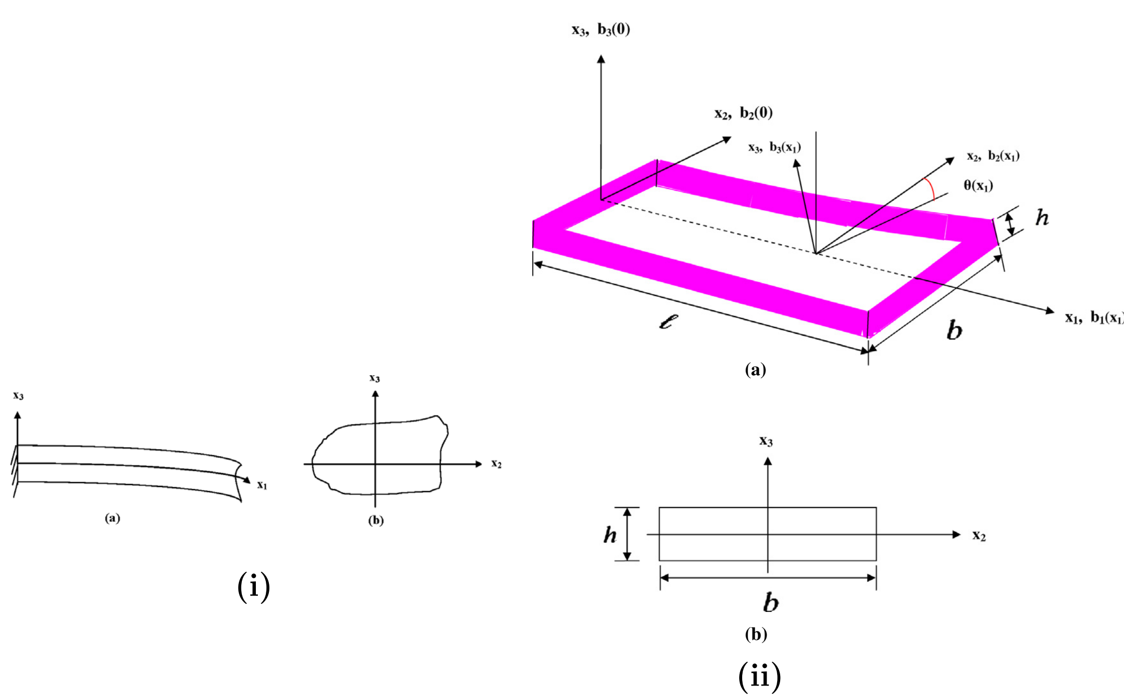

Fig. 1. Sketch of (i-a) initially twisted and curved cantilever beam, (i-b) its cross-section and (ii-a) initially twisted strip-like beam configuration and coordinate system and (ii-b) its cross-section.