Engineering Structures 110 (2016) 169–183

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

New first order shear deformation beam theory with in-plane shear influence

Ivo Senjanović *, Nikola Vladimir, Neven Hadžić, Marko Tomić

University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture, Ivana Lucica 5, 10000 Zagreb, Croatia

ARTICLE INFO

Article history:
Received 3 July 2015
Revised 20 October 2015
Accepted 30 November 2015
Available online 29 December 2015

Keywords:
Modified Timoshenko beam theory
First order shear deformation theory
New deformation concept
Flexural and in-plane shear vibrations

ABSTRACT

In the conventional formulation of the Timoshenko beam theory bending and shear deflection cannot be determined uniquely. Recently, an alternative formulation which deals with total deflection and bending deflection has been developed with unique results. In the present paper a new beam theory, taking coupling between flexural and in-plane shear vibrations into account, is derived by employing Hamilton's principle. First, uncoupled flexural and in-plane shear vibrations are considered. Then, coupling of vibrations, manifested in the case of geometric boundary conditions, is realized in a physically transparent way. Accuracy is confirmed by 2D FEM vibration analysis of illustrative examples. The proposed formulation is superior to the known first order shear deformation beam theories.

1. Introduction

Beam is a constitutive structural element in many engineering structures. For slender beam the Euler–Bernoulli theory is usually used, but for thick beam the Timoshenko beam theory is preferable since it takes in addition both shear and rotary inertia into account [1,2]. The theory is a simplification of real beam behavior, with linear approximation of the parabolic shear stress distribution across beam cross-section. Shear effect is taken into account by shear coefficient as relation parameter between shear stress and shear strain.

General beam theories are developed, where beams are subjected to bending, torsion and tension, with shear influence on deflection and stresses. Three recently published books give a good review of those theories [3–5]. Complex problem of coupled flexural and torsional vibrations of beam-like structures as ship hull with multi-cell cross-section and variable properties is presented in [6,7].

In order to achieve a realistic beam behavior many higher-order shear deformation theories have been developed, assuming curvilinear beam cross-section deformation for instance [8,9]. However, great efforts resulted with small effect on accuracy, and therefore the first order shear deformation theory is still in practical use. It is subject of many investigations in order to increase its efficiency.

In ordinary beam static analysis, bending and shear deflection are treated as two basic physical quantities. Total deflection is obtained by summing up bending and shear deflection. In case of clamped and combined clamped and simply supported end, bending and shear deflection do not satisfy boundary condition individually, while the total deflection does. Based on expected physical reality, each of beam deflection should satisfy geometric boundary conditions. This problem is not recognized in the Timoshenko beam theory [10]. Actually, it is discussed by Donnel [11], but it is ignored as an effect of rigid body motion.

In vibration analysis by the Timoshenko beam theory bending and shear deflection cannot be determined uniquely. Therefore, a few improved theories have been developed. For instance, bending and shear deflection are determined independently in [12], and total deflection is obtained as their summation. Governing differential equations and boundary conditions are successfully derived in [13] by employing Hamilton's principle. However, both differential equations and boundary conditions are the same as those in the conventional theory. Hence, there is no a new achievement in this case.

Recently, an alternative formulation of the boundary value problem for the Timoshenko beam has been proposed [14,15]. Bending and shear deflection are adopted as independent physical entities. Total deflection and bending deflection, which causes cross-section rotation, are chosen as basic variables. In such a way bending and shear deflection are uniquely determined. This theory gives somewhat higher values of natural frequencies in case of geometric boundary conditions. However, reliability of the obtained results is not checked by a more sophisticated approach. There is doubt should all three deflections, i.e. total, bending and shear, and cross-section rotation be constrained, since in a physical model test only visual beam displacements, i.e. total deflection and cross-section rotation can be fixed. Bending and shear deflection are internal quantities which actually cannot be influenced by outside.

* Corresponding author.
E-mail address: ivo.senjanovic@fsb.hr (I. Senjanović).

http://dx.doi.org/10.1016/j.engstruct.2015.11.032
0141-0296/

Motivated by the above state-of-the art in this paper an extensive investigation of beam dynamic behavior is undertaken as a continuation of the authors' earlier paper [16]. New concept of displacements and sectional forces is introduced. Additionally to the flexural parameters, in-plane shear cross-section rotation and corresponding moment are taken into account. Coupling phenomenon between flexural and in-plane shear vibrations is recognized in case of geometric boundary conditions. The obtained results are checked by 2D FEM vibration analysis.

2. Outline of the classical beam theory

Traditional Timoshenko beam theory deals with beam deflection and cross-section rotation, w and ψ, respectively, Fig. 1, [1,2]. The sectional forces, i.e. bending moment and shear force, read

$$M = -D\frac{\partial \psi}{\partial x}, \quad Q = S\left(\frac{\partial w}{\partial x} - \psi\right), \tag{1}$$

where $D = EI$ is flexural rigidity and $S = kGA$ is shear rigidity, A is cross-section area and I is its moment of inertia, k is shear coefficient, and E and $G = E/[2(1+\nu)]$ is Young's modulus and shear modulus, respectively.

Beam is loaded with transverse inertia load per unit length and distributed inertia moment

$$q_x = -m\frac{\partial^2 w}{\partial t^2}, \quad m_x = J\frac{\partial^2 \psi}{\partial t^2}. \tag{2}$$

where $m = \rho A$ is specific mass per unit length and $J = \rho I$ is its moment of inertia.

Beam strain energy and kinetic energy are the following:

$$\begin{aligned} U &= \frac{1}{2}D\int_0^l \left(\frac{\partial \psi}{\partial x}\right)^2 \mathrm{d}x + \frac{1}{2}S\int_0^l \left(\frac{\partial w}{\partial x} - \psi\right)^2 \mathrm{d}x, \\ K &= \frac{1}{2}m\int_0^l \left(\frac{\partial w}{\partial t}\right)^2 \mathrm{d}x + \frac{1}{2}J\int_0^l \left(\frac{\partial \psi}{\partial t}\right)^2 \mathrm{d}x. \end{aligned} \tag{3}$$

The governing equilibrium equations and boundary conditions can be derived by employing Hamilton's principle

$$\delta\int_{t_1}^{t_2} (K - U)\mathrm{d}t = 0, \tag{4}$$

where δ denotes variation. Hence, one obtains

$$\begin{aligned} &D\frac{\partial^2 \psi}{\partial x^2} + S\left(\frac{\partial w}{\partial x} - \psi\right) - J\frac{\partial^2 \psi}{\partial t^2} = 0, \\ &S\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \psi}{\partial x}\right) - m\frac{\partial^2 w}{\partial t^2} = 0, \end{aligned} \tag{5}$$

$$\left[D\frac{\partial \psi}{\partial x}\delta\psi\right]_0^l = 0, \quad \left[S\left(\frac{\partial w}{\partial x} - \psi\right)\delta w\right]_0^l = 0. \tag{6}$$

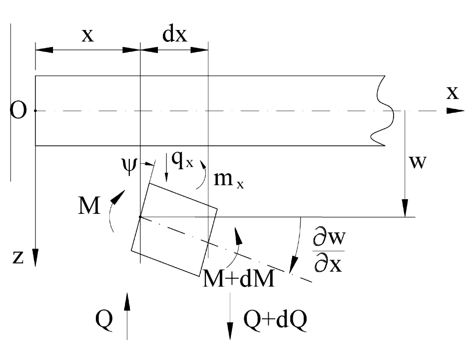

Fig. 1. Classical concept of beam displacements and sectional forces.

The second of Eqs. (5) yields

$$\frac{\partial \psi}{\partial x} = \frac{\partial^2 w}{\partial x^2} - \frac{m}{S}\frac{\partial^2 w}{\partial t^2}. \tag{7}$$

By substituting (7) into the first of (5) one arrives at [17]

$$\frac{\partial^4 w}{\partial x^4} - \left(\frac{m}{S} + \frac{J}{D}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{m}{D}\frac{\partial^2}{\partial t^2}\left(w + \frac{J}{S}\frac{\partial^2 w}{\partial t^2}\right) = 0. \tag{8}$$

Once (8) is solved rotation angle is determined from (7)

$$\psi = \frac{\partial w}{\partial x} - \frac{m}{S}\int \frac{\partial^2 w}{\partial t^2}\mathrm{d}x + f(t), \tag{9}$$

where $f(t)$ is a rigid body rotational motion.

3. New beam theory

The traditionally used Timoshenko beam theory deals with two visible beam displacements, i.e. deflection w and cross-section rotation ψ. Since shear is present, it is obvious that total deflection consists of bending deflection and shear deflection, w_b and w_s, respectively. They are not visible, but can be constructed based on the known w and ψ.

In the new beam theory it is assumed that cross-section rotation includes bending angle, φ, and possible in-plane shear angle, ϑ. If ϑ is uniform along the beam, that associates on sheared set of playing cards. Hence, according to Fig. 2a, one can write for the total displacements

$$\begin{aligned} &w = w_b + w_s, \\ &\psi = \varphi + \vartheta, \quad \varphi = \frac{\partial w_b}{\partial x}, \\ &\mu = \varphi + \lambda, \quad \lambda = \frac{\partial w_s}{\partial x}, \end{aligned} \tag{10}$$

where λ is shear angle and μ is rotation of beam neutral axis.

Distributed transverse load q_x acting on a beam causes bending and transverse shear with sectional bending moment, $M_b = -Dw_b''$, and shear force, $Q_s = Sw_s'$, Fig. 2b. Distributed rotary load m_x also causes bending and shear, with corresponding bending moment, $M^* = -Dw_b^{*''}$, and shear force, $Q^* = Sw_s^{*'}$. Some elementary static examples are presented in [4,18].

In case of a very short and high beam, like membrane with large aspect ratio h/l, in-plane shear deformation occurs and it is dominant. The corresponding moment reads $M^* = -D\vartheta'$, where ϑ is in-plane shear angle, Fig. 2c. This problem is recognized in beam higher natural modes where each segment between adjacent vibration nodes represents actually a short beam.

In accordance with the above consideration the total sectional forces read

$$M = M_b + M^* = -D\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial \vartheta}{\partial x}\right), \quad Q = S\left(\frac{\partial w_s}{\partial x} - \vartheta\right) = Q_s + Q^*, \tag{11}$$

where $Q^* = -S\vartheta$ can be interpreted as shear force due to in-plane shear, for the time being, Fig. 2c.

Strain and kinetic energy of the considered beam are the following

$$U = \frac{1}{2}D\int_0^l \left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial \vartheta}{\partial x}\right)^2 \mathrm{d}x + \frac{1}{2}S\int_0^l \left(\frac{\partial w_s}{\partial x} - \vartheta\right)^2 \mathrm{d}x, \tag{12}$$

$$K = \frac{1}{2}m\int_0^l \left(\frac{\partial w_b}{\partial t} + \frac{\partial w_s}{\partial t}\right)^2 \mathrm{d}x + \frac{1}{2}J\int_0^l \left(\frac{\partial^2 w_b}{\partial x \partial t} + \frac{\partial \vartheta}{\partial t}\right)^2 \mathrm{d}x. \tag{13}$$