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A general extended Kalman filter for simultaneous estimation of system and unknown inputs

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ABSTRACT

The traditional Extended Kalman filter (EKF) is a useful tool for structural parameter identification with limited observations. It is, however, not applicable when the excitations on the structure are unknown or the excitation locations are not monitored. A novel Extended Kalman filter approach referred to as the General Extended Kalman filter with unknown inputs (GEKF-UI) is proposed to estimate the structural parameters and the unknown excitations (inputs) simultaneously. The proposed GEKF-UI gives an analytical EKF solution dealing with the more general measurement scenarios with the existing EKF methods as its special cases. Existing constraints on sensor configuration have been removed enabling more general application to complex structures. Simulation results from a 3-storey linear damped shear building, an ASCE benchmark structure and a two-storey planar frame structure are used to validate the proposed method for both time-invariant and time-varying system identification.

1. Introduction

1.1. The simultaneous state and unknown input estimation using Kalman filtering

The Extended Kalman filter (EKF) [1] is a simple but powerful tool for the identification of structural parameters such as stiffness, damping and nonlinear parameters with limited measurements for structural health monitoring (SHM) of structures [2–5]. Existing EKF approaches treat the unknown structural parameters as part of the states to be estimated. To track the evolution of the estimates, all the external excitations (inputs) should be known or measured. However, some inputs may not be measured or known in practice, such as the seismic excitations, the ambient wind loads, the moving traffic loads, etc. Therefore, an EKF approach which can deal with unknown inputs is in need. Though theoretical study on linear filtering with unknown inputs has been extensively studied during the past few decades [6–12], yet their nonlinear counterparts, the EKF method with unknown inputs have not been broadly investigated. Wang and Haldar [13] proposed an iterative least-squares procedure in conjunction with the traditional EKF to estimate the states and unknown inputs off-line with limited observations. Lately, Al-Hussein and Haldar [14] improved the method by replacing the EKF formulation with the more recent unscented Kalman filter (UKF) to obtain improved performance of structural health assessment (SHA). However, the drawbacks in the numerical procedure of the off-line original method [13] are still retained. To track the evolution of the structural parameters (damages of the structures) on-line, an extended Kalman filter with unknown inputs (EKF-UI) was derived analytically by minimizing a least-squares objective function [15]. More recently, a method which sequentially estimates the unknown inputs with the least-squares estimation and the states with the traditional EKF was proposed [16,17] for both linear and nonlinear structures. Other than the identification of external excitations, it has been shown that the EKF with unknown inputs can be extended to a large structural system [16] based on a decomposition method. The intra-connection effect between structural elements is regarded as unknown input to the substructures. The measurements at the sub-structural interface degrees-of-freedom (DOFs) are not necessary as a result, and the number of sensors in the SHM system for a large scale structure is thus reduced. These serve as good justifications for the development of a new EKF approach with unknown inputs.

For the above-mentioned EKF approaches [13–16], all inputs are required to be present in the observation equations, which imposes constraints on the number and location of the sensors required. For

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instance, the sensors should be installed at all the DOFs corresponding to the external excitations in Lei et al. [16]. Moreover, if the measurements are not in the form of accelerations, extra signal processing is needed with additional computation effort. For example, before the application of the EKF-UI approach [15], double numerical differentiations have to be made with the aid of low-pass digital filter to obtain the accelerations from the measured displacements of a 3-storey nonlinear shear building subjected to unknown earthquake. To reduce these restrictions Pan et al. [18] have proposed an EKF with unknown inputs without direct feed-through (EKF-UI-WDF) approach where all the unknown inputs are in the model equations whereas none of them are explicitly shown in the observation equations.

1.2. Weakness with existing Kalman filtering approaches with unknown inputs

To the best of the authors' knowledge, literature on Kalman Filter with unknown inputs is mainly divided into two categories: (I) those with the unknown inputs included in both the model equations and the observation equations [9,11–17]; and (II) those with the unknown inputs found only in the model equations [6,8,10,18]. The coefficient matrix of unknown inputs in the observation equations of the first group should be of full column rank [15]. The same coefficient matrix for the latter group is a zero matrix, which results in one step delay in the estimation of the unknown inputs.

To demonstrate the differences more clearly, an ASCE benchmark 4-storey structure [19] subjected to two white noise excitations at the 2nd and 4th floors is adopted in the following discussion. Numerical analysis on this example is given in Section 4 of this paper.

By introducing the extended state vector $\mathbf{Z} = \{\mathbf{x}^{\mathrm{T}}, \dot{\mathbf{x}}^{\mathrm{T}}, \theta^{\mathrm{T}}\}^{\mathrm{T}}$ into the equations of motion of the structure, the model equation becomes $\dot{\mathbf{Z}}(t) = \mathbf{g}(\mathbf{Z}, \mathbf{f}^*, t)$ where $\mathbf{x}, \dot{\mathbf{x}}, \theta$ are vectors of the storey displacements, velocities and unknown structural parameters, respectively. $\mathbf{f}^*(t) = [f_2(t) f_4(t)]^{\mathrm{T}}$ is the unknown input vector. Cases with different measurement scenarios are discussed as follows:

Case (i) When the measurements are accelerations from all stories at $t = (k+1)\Delta t$ ($k = 0, 1, 2, \ldots$), i.e., $\mathbf{y}_{k+1} = [\ddot{x}_{1,k+1}\ddot{x}_{2,k+1}\ddot{x}_{3,k+1}\ddot{x}_{4,k+1}]^{\mathrm{T}}$. The observation equations become $\mathbf{y}_{k+1} = \mathbf{h}(\mathbf{Z}_{k+1}, \mathbf{f}^*_{k+1}, k+1)$ where $\mathbf{f}^*_{k+1} = [f_{2,k+1} f_{4,k+1}]^{\mathrm{T}}$. Since the rank of the coefficient matrix with unknown inputs $\mathbf{D}^*_{k+1|k} = [\partial\mathbf{h}_{k+1}/\partial\mathbf{f}^*_{k+1}] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}$ is two and it is of full column rank, the condition of estimation [14] holds. Therefore, Category I EKF approaches are applicable for the solution [15–17].

Case (ii) When the measurements are displacements from all stories at $t = (k+1)\Delta t$ ($k = 0, 1, 2, \ldots$), i.e., $\mathbf{y}_{k+1} = [x_{1,k+1}x_{2,k+1}x_{3,k+1}x_{4,k+1}]^{\mathrm{T}}$. The observation equations become $\mathbf{y}_{k+1} = \mathbf{h}(\mathbf{Z}_{k+1}, k+1)$ where no unknown input exists. The rank of $\mathbf{D}^*_{k+1|k} = [\partial\mathbf{h}_{k+1}/\partial\mathbf{f}^*_{k+1}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$ is zero and is not full rank, and the condition of estimation [15] does not hold. The Category I EKF approaches are not applicable as a result while Category II approaches [18] can deal with this case.

Case (iii) When no sensor is installed on the second floor, i.e., $\mathbf{y}_{k+1} = [x_{1,k+1}x_{3,k+1}x_{4,k+1}]^{\mathrm{T}}$, the rank of $\mathbf{D}^*_{k+1|k} = [\partial\mathbf{h}_{k+1}/\partial\mathbf{f}^*_{k+1}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}$ is unity which is less than full rank. Therefore, both Categories of EKF approaches discussed above are not applicable. In other words, the simultaneous estimation of the states and inputs with this scenario of measurement cannot be solved with any existing EKF approaches.

As a summary,

(a) For the simultaneous estimation of the state and unknown inputs, Cases (i) and (ii) above show that the EKF approaches in different forms are needed for different measurement scenarios, which brings unnecessary inconvenience for the computation.
(b) Case (iii) shows that for a general measurement scenario where only partial inputs are present in the observation equations, with $0 < rank(\mathbf{D}^*_{k+1|k}) < r$, where r is the full column rank, no existing EKF approaches is applicable.

In this paper, a novel EKF approach referred to as the General EKF with unknown inputs (GEKF-UI) is proposed to identify structural parameters and unknown inputs simultaneously. It is an EKF approach with partial measurement and unknown inputs covering the case of $0 \leqslant rank(\mathbf{D}^*_{k+1|k}) \leqslant r$, which is more general than existing approaches that require $rank(\mathbf{D}^*_{k+1|k}) = r$ or $rank(\mathbf{D}^*_{k+1|k}) = 0$. There is no need to measure the responses at all of the DOFs on which the unknown inputs are acting as a result.

The recursive solutions of the GEKF-UI are formulated with the least-squares estimation of an extended state vector with the aid of matrix decompositions. The unknown inputs at all time instants and the current states are combined as the extended state vector whose dimension increases with time. The relationship between the extended state vectors at two consecutive time instants is derived to form a recursive solution on the extended state vector. Matrix decomposition is then applied to obtain the extended state vector.

Simulation results from the following examples are used to validate the proposed method: (a) a three-degrees-of-freedom (DOFs) linear damped shear building subjected to an unknown earthquake excitation; (b) the Phase I ASCE benchmark building for structural health monitoring subjected to two unknown white noise excitations; and (c) a 2-storey plane frame with 12 DOFs subjected to unknown inputs with abrupt reduction of stiffness.

The layout of this paper is given as follows. The problem formulation is given in Section 2. The recursive solutions for the proposed GEKF-UI approach are derived and presented in Section 3 with discussions. To verify the proposed method, three numerical examples are presented in Section 4. Section 5 gives the conclusions. Detailed derivations on the solutions in Sections 3 are given in Appendices A and B.

2. Problem formulation

When the external excitations are not measured (unknown), the equations of motion of a m-DOFs linear damped structure can be expressed as:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{F}_c[\dot{\mathbf{x}}(t)] + \mathbf{F}_s[\mathbf{x}(t)] = \boldsymbol{\eta}^*\mathbf{f}^*(t) + \boldsymbol{\eta}\mathbf{f}(t) \tag{1}$$

where $\mathbf{f}^*(t) = [f_1^*(t), \ldots, f_r^*(t)]^{\mathrm{T}}$ is the set of r-unknown (or unmeasured) excitations; Matrix $\boldsymbol{\eta}^*$ is a mapping matrix associated with $\mathbf{f}^*(t)$; $\mathbf{f}(t) = [f_1(t), f_2(t), \ldots, f_s(t)]^{\mathrm{T}}$ is the set of s-known (measured) excitations; and $\boldsymbol{\eta}$ is a $(m \times s)$ mapping matrix associated with $\mathbf{f}(t)$.

Consider an extended unknown state vector with a dimension of $(2m+n)$,

$$\mathbf{Z}(t) = \{\mathbf{x}^{\mathrm{T}}, \dot{\mathbf{x}}^{\mathrm{T}}, \theta^{\mathrm{T}}\}^{\mathrm{T}} \tag{2}$$