Engineering Structures 103 (2015) 15–27

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Experimental and finite element dynamic analysis of incrementally loaded reinforced concrete structures

Ninoslav Pešić [a,*], Stana Živanović [a], Jamie Dennis [b], James Hargreaves [b]

[a] *University of Warwick, School of Engineering, Coventry CV4 7AL, United Kingdom*
[b] *ARUP, Adv. Technology & Research Division, Blythe Gate, Blythe Valley Park, Solihull B90 8AE, United Kingdom*

ARTICLE INFO

Article history:
Received 11 September 2014
Revised 4 June 2015
Accepted 21 July 2015
Available online 9 September 2015

Keywords:
Concrete
Modal testing
FE analysis
Structural dynamics
Damage assessment

ABSTRACT

This work investigates influence of damage in reinforced concrete (RC) structures on their dynamic properties through modal testing and non-linear finite element (FE) analysis. Five RC beams were designed with the fundamental flexural mode frequencies in the range of 6.5–18.0 Hz for the uncracked state. Mechanical properties of concrete, such as static and dynamic elastic moduli were determined from standard tests and ultra-sonic pulse velocity readings. The beams were incrementally loaded until the span/250 deflection limit was reached and their natural frequencies were measured from the free decay vibrations. The progressive damage reduced fundamental frequencies of tested beams by up to 25%. The non-linear FE analysis was carried out for RC beams and one two-span slab and the calculated reduced frequencies of the 1st and 2nd vibration modes were in excellent agreement with measurements. This led to the conclusion that, given that the non-linear analysis can capture degradation of dynamic stiffness due to cracking, the future dynamic performance and damage identification on the RC structure can be reliably determined from the same FE model. The results reveal potential of the combined modal testing and FE analysis to improve inspection and assessment of the in-service RC structures.

1. Introduction

1.1. Background

The modal analysis of the in-service RC structures is gaining prominence as non-destructive damage assessment technique from the basic premise that the measured reduction of natural frequencies of vibration serves as a structural health or damage indicator. This concept brings together traditional inspection methodology, the experimental modal analysis and structural dynamic analysis for investigating structural performance and/or damage in the existing RC structures.

Among the most important parameters in the dynamic analysis and integrity assessment of any RC structure is the value of the modulus of elasticity for concrete. The distinction between the *static* and *dynamic* modulus of elasticity, both depending on the concrete mix, has already been made from small specimens and beam tests [1–3]. The static modulus, E_c, is determined from the standardised cylinder compression tests while the dynamic modulus, $E_{c,dyn}$, is obtained from the resonance tests or the ultra-sonic pulse velocity (UPV) readings [4]. Several empirical equations correlate compressive strength and static modulus of concrete to the dynamic modulus [5,6]. The dynamic modulus of concrete is typically 10% to 40% larger than the static modulus [7].

It will be addressed in the course of our analysis whether the value of the static or dynamic elastic modulus of concrete should feature in the governing equation of flexural vibrations of RC beams. In the case of a simply supported *cracked* RC beam with the cross-sectional area and the second moment of area, $A(x)$ and $I(x)$, varying along the span, L_s, this equation has the form:

$$\frac{\partial^2}{\partial x^2}\left[E_{c/c,dyn}I(x)\frac{\partial^2 y}{\partial x^2}\right]+m_b A(x)\frac{\partial^2 y}{\partial t^2}=0 \tag{1}$$

Here, y is the sectional displacement (deflection), x is the section coordinate along the beam span ($0 \leqslant x \leqslant L_s$), m_b is the mass per unit length of the beam and t denotes time. To calculate the frequency of the n-th mode of vibration, the closed-form solution to Eq. (1) is readily available for the simplest case when the beam cross-section is assumed to be uniform along the span:

$$f_{nc}=\frac{\pi}{2}\left(\frac{n}{L_s}\right)^2\sqrt{\frac{E_{c/c,dyn}I_{ec}}{m_b}} \qquad (n=1,2,3,\ldots) \tag{2}$$

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[*] Corresponding author.
E-mail address: n.pesic@warwick.ac.uk (N. Pešić).
URL: http://www2.warwick.ac.uk/fac/sci/eng/ (N. Pešić).

http://dx.doi.org/10.1016/j.engstruct.2015.07.037
0141-0296/

In Eq. (2), the flexural stiffness of the cracked RC beams is product of the elastic modulus of concrete and the effective second moment of area defined by the Eurocode 2 [8,9] as:

$$I_{ec} = \alpha I_{c1} + (1 - \alpha) I_{c2} \tag{3}$$

where I_{c1} and I_{c2} are the second moment of areas of the gross and cracked RC section, respectively, while $0.60 \leqslant \alpha \leqslant 0.80$ is the load parameter [9]. This approximation of the second moment of area might be suitable for estimating the frequency reduction on RC beams at the design stage but is not applicable for damage identification on the more complex structural elements or when the cumulative effects of incremental loading need to be assessed.

In an improvement over Eq. (3), a continuous function for reduced flexural stiffness was proposed for damage identification on cracked RC beams with known frequencies [10]. Three damage parameters, denoted as α, β and n represent reduced stiffness of the cross-section, the length and the shape of the beam damaged zone and are iteratively determined by equating the measured frequencies with those calculated from the 2D segmental model of the beam. These analysis steps outline the basics of the model updating technique that needs to be expanded for dynamic analysis of geometrically more complex RC structures warranting finite element (FE) modelling and analysis.

It was observed from the experimental modal analysis of the incrementally damaged RC beams and slabs [8,11–13] that their frequency of the 1st vibration mode can be reduced up to 30% when the yielding of the tensile reinforcement occurs [14,15]. Such magnitude of frequency reduction, being a consequence of structural damage, gave rise to the concept of damage identification by means of modal testing [16,17]. The method is applicable to RC structures for a range of problems from vibration serviceability of the concrete floor slabs in buildings to the assessment of bridges.

The FE model updating technique has already been used for accurate evaluation of the dynamic properties of the field-tested concrete structures that exhibit linear dynamic behaviour [18]. This work will further examine the benefits of simultaneous use of the experimental modal analysis and non-linear FE analysis for the structural assessment of traditional RC beams, slabs or other types of structures whose natural frequencies inevitably reduce due to the load-induced cracks in concrete. As cracking introduces non-linear response of RC structures, their structural performance and dynamic behaviour cannot be accurately assessed nor predicted by means of linear analysis [19,20].

1.2. Research significance and objectives

The case now exists to expand the FE model updating methodology to the non-linear analysis of cracked RC beams and more complex structural elements modelled with solid 2D/3D or shell elements. Once sufficiently accurate natural frequencies are computed from the updated FE models, zones with structural damage could be identified from the same models using parameters that define the non-linear constitutive law for concrete.

The concept will be studied through the modal testing of five RC beams with different span/depth ratios and through the non-linear FE analysis of their 2D/3D models. The investigated points are:

- time-development of the concrete strength and static and dynamic modulus of elasticity;
- the rate of frequency reduction in RC beams with the increase in the applied loading;
- influence of the number of load cycles on the frequency reduction in RC beams;
- the accuracy of the FE analysis in predicting natural frequencies of cracked RC beams and identification of the load-induced damage in concrete.

For further correlation of results from the experimental modal testing and dynamic analysis with the extent of damage in RC structures, the non-linear analysis will also be performed on the FE models of an independently tested two-span RC slab [12].

2. Experimental programme

Five RC beams were incrementally loaded and their modal properties determined after each load-unload step. This section reports on the mechanical properties of concrete, the experimental configuration of RC beams and, finally, the results from their modal testing.

2.1. Properties of concrete

Details of the concrete mix and the curing regime are provided in Table 1. Compressive strength of concrete was evaluated as the average from three 100 mm cubes crushed in the period from 7 to 250 days after casting. Cylinder compressive strength was obtained at the age of concrete of 32 and 105 days and the static elastic modulus at the age of 32 days (this is comparable to the 28-days reference age of concrete in codes of practice). Cement CEM-II/B-V contained 30% fly-ash which, due to its slower hydration rate, added to the modest compressive strength gain after the age of 20 days. Time development of the concrete compressive strength until the age of 250 days is shown in Fig. 1. The 32-day characteristic cube and cylinder compressive strengths ($f_{ck,cube} \approx 48$ MPa and $f_{ck} \approx 43$ MPa) place this concrete between the “C35/45” and “C40/50” Eurocode 2 classes [9].

Using the averaged UPV readings, v_p, on 300, 400 and 500 mm long prisms with the same cross-sections as the tested beams (Section 2.2), the dynamic modulus of concrete was evaluated from the following relation:

$$E_{c,dyn} = \rho_c \, v_p^2 \, \frac{(1 - 2\vartheta)(1 + \vartheta)}{1 - \vartheta} \tag{4}$$

in which the Poisson’s coefficient was adopted as a constant $\vartheta = 0.20$ while the density of concrete cubes varied with time (due to drying) within the range of $2300 \geqslant \rho_c \geqslant 2250\ \text{kg/m}^3$.

The static modulus of elasticity was experimentally determined as $E_c \approx 30.5$ GPa. The Eurocode 2 equation $E_c = 22.0(f_{cm}/10)^{0.30}$ [GPa], in which $f_{cm} = f_{ck} + 8$ [MPa] is the mean characteristic compressive strength of concrete, over-estimates the experimental E_c value by nearly 20%. A much better estimate of the static modulus was made using the empirical formula of Noguchi et al. [21]:

$$E_c = 33.5\; k_1\, k_2\, (\rho_c/2400)^2\, (f_{cm}/60)^{1/3} \qquad [\text{GPa}] \tag{5}$$

Table 1
Details of the concrete mix.

Material into 1 m^3 of concrete volume	Mass [kg]	Volume [m^3]
Cement CEM II/B-V + SR	380	0.140
Aggregate		
0–4 mm (quartz)	895	0.338
4–20 mm (quartzite)	895	0.350
Water (W/C = 0.44)	168	0.168
Plasticiser CSP340	2.0	0.002
Air content (estimated)	/	0.002

Indoor curing, humidity = 55%:
wet hessian applied for 14 days only.