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Improved accuracy for the C_m factor of steel beam-columns

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ABSTRACT

The C_m factor acting originally as equivalent moment factor for stability checking of beam-columns, assumes an additional function as part of the magnification factor B_1 since the AISC–LRFD specification adopted a single equation for beam-column strength calculation. This article first clarifies the difference between the two roles of C_m for in-plane instability checking and section yielding checking and establishes the relationship between the two C_m. As there is no distinction between the two C_m, some confusion occurs in the current AISC specification, and C_m as equivalent moment factor is not accurately evaluated in many cases. In the present study, the ratio P/P_e is introduced into the C_m formula for beam-columns subject to end moments to enhance its precision. For members subject to transverse loading, formulas based on rational analysis are given in order to avoid the use of conservative value of 1.0. By making use of the principle of superposition, simple and accurate formulas are obtained for the case of transverse loading combined with end moments. The proposed formulas are compared with the calculation by finite element method to ensure their feasibility for designing elastic–plastic members with initial imperfections. The results are satisfactory.

1. Introduction

In establishing the formula for strength checking of beam-columns, the bending moment is always assumed as uniformly distributed, whereas in actual members, the bending moment often varies along the member axis. This discrepancy is compensated by a factor C_m. which, for a long period of time, is roughly evaluated. An obvious fact is that both the code for structural concrete ACI 318-11 [1] and the specification for structural steel AISC 360-10 [2] prescribe $C_m = 1.0$ for beam-columns subjected to transverse loading.

Pallarés et al. [3] analyzed beam-columns of normal and high strength concrete, and proposed a series of expressions of the C_m factor, improving the level of safety and accuracy of member design. However, owing to the difference in material behavior, the findings of Pallarés et al. cannot be used to steel structures.

The objective of this study is to derive simple and accurate formulas of the C_m factors of steel beam-columns under various loading condition. The results obtained are feasible for use in the design of both regular steel members and cold-formed steel members.

2. A historical overview

The C_m factor appeared early in the formula for checking the stability of beam-columns in the AISC–ASD specification for structural steel buildings (AISC, 1978) [4]. For the sake of simplicity, the relevant formula is given here for members under uniaxial bending, i.e.,

$$\frac{f_a}{F_a} + \frac{C_m f_b}{\left(1 - \frac{f_a}{F'_e}\right) F_b} \leqslant 1.0 \tag{1}$$

where F_a = axial compressive stress that would be permitted if axial force alone existed; F_b = compressive bending stress that would be permitted if bending moment alone existed; F'_e = Euler stress divided by a factor of safety, and taking account of the effective length factor K; f_a = computed axial stress; and f_b = computed compressive bending stress at the point under consideration.

The factor C_m is a multiplier to the bending stress, aiming to transform the effect of non-uniform bending into that of an equivalent uniform moment. In this sense, C_m is commonly called as "equivalent moment factor".

Parallel with Eq. (1), there is another formula for checking the section yielding, which is irrelevant to C_m.

The AISC–LRFD specification for structural steel buildings (AISC, 1993) [5] has two major amendments regarding the strength calculation of beam-columns. Firstly the calculation is based on

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second-order stresses in lieu of first-order ones. Secondly, the two formulas of strength checking merge into one bilinear equation, i.e.,

$$\text{For} \quad \frac{P_u}{\phi_c P_n} \geqslant 0.2 \qquad \frac{P_u}{\phi_c P_n} + \frac{8M_u}{9\phi_b M_n} \leqslant 1.0 \tag{2a}$$

$$\text{For} \quad \frac{P_u}{\phi_c P_n} < 0.2 \qquad \frac{P_u}{2\phi_c P_n} + \frac{M_u}{\phi_b M_n} \leqslant 1.0 \tag{2b}$$

where P_u = required compressive strength; P_n = nominal compressive strength for flexural buckling of compression member; M_u = required flexural strength determined from a second-order elastic analysis; M_n = nominal flexural strength for beams; ϕ_c = resistance factor for compression; and ϕ_b = resistance factor for flexure.

The second-order bending moment M_u may be approximated by the following linear equation

$$M_u = B_1 M_{nt} + B_2 M_{lt} \tag{3}$$

where M_{nt} = required flexural strength in member assuming there is no lateral translation of the frame; M_{lt} = required flexural strength in member as a result of lateral translation of the frame only.

$$B_1 = \frac{C_m}{1 - P_u/P_{e1}} \geqslant 1 \tag{4}$$

where $P_{e1} = A_g F_y/\lambda_c^2$, in which λ_c = the slenderness parameter, $\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}}$. For simply supported beam-columns, $K = 1$ and $P_{e1} = P_e$, the Euler load.

$$B_2 = \frac{1}{1 - \sum P_u(\Delta_{oh}/\sum HL)} \tag{5}$$

where Δ_{oh} = lateral inter-story deflection; $\sum H$ = sum of all story horizontal forces producing Δ_{oh}; and L = story height.

The C_m factor this time has no relevance to the bending moment caused by joint translation. But, on the other hand, as Eq. (2) has the dual functions of checking member stability and section yielding, C_m has also to play two roles: the one is the equivalent factor for stability calculation, and the other is a constituent part of the amplification factor B_1 for section yielding checking. The dual functions of C_m should be clarified in the specification and their evaluation approach distinguished. Otherwise, there will inevitably be some confusion in the comprehension of the factor.

The unified AISC ASD/LRFD specification for structural steel buildings (AISC, 2010) [2] introduces a new method of stability design namely the direct analysis method. This new method, with consideration of initial imperfections and adjustments to stiffness, can guarantee the overall stability of the structure through analysis. But it is still necessary to check member stability. For beam-columns, the relevant formula is basically the same as Eq. (2), but with different notations.

$$\text{For} \quad \frac{P_r}{P_c} \geqslant 0.2 \qquad \frac{P_r}{P_c} + \frac{8M_r}{9M_c} \leqslant 1.0 \tag{6a}$$

$$\text{For} \quad \frac{P_r}{P_c} < 0.2 \qquad \frac{P_r}{2P_c} + \frac{M_r}{M_c} \leqslant 1.0 \tag{6b}$$

where P_c and M_c are equivalent to $\phi_c P_n$ and $\phi_b M_n$ of Eq. (2) respectively; P_r and M_r are equivalent to P_u and M_u respectively. When M_r is determined by rigorous second-order analysis, it has nothing to do with the C_m factor. But the specification also allows the use of the approximate moment of second-order analysis, identical to AISC 1993 and represented by Eqs. (3)–(5). When this latter approach is adopted, the C_m factor remains useful and has dual functions to perform as before.

On the other hand, a new formula for separate checking of the out-of-plane bucking is introduced in the unified specification, namely

$$\frac{P_r}{P_{cy}}\left(1.5 - 0.5\frac{P_y}{P_{cy}}\right) + \left(\frac{M_{rx}}{C_b M_{cx}}\right)^2 \leqslant 1.0 \tag{7}$$

where P_r and M_{rx} = required axial strength and flexural strength respectively; C_b = lateral-torsional bucking modification factor; and M_{cx} = available lateral-torsional strength for strong axis flexure calculated with $C_b = 1.0$.

The C_m factor, inherent in the term M_{rx}, is supposed to act as an equivalent moment factor for out-of-plane bucking. However, the factor C_b performs the same function so that C_m is a duplicate of $1/C_b$. Therefore, for a beam-column without end translation, the C_m factor in the expression

$$M_{rx} = B_1 M_x = \frac{C_m M_x}{1 - P_r/P_{e1}} \tag{8}$$

should be taken as unity for Eq. (7). Thus, C_m still has two functions same as in the LRFD specification. As there is no clarification in this regard, an obvious confusion about the naming of C_m occurs in the current AISC specification (AISC, 2010). Four different definitions of C_m coexist in the body and the commentary of the specification, they are:

- "*Coefficient accounting for nonuniform moment*" in the list of symbols
- "*Equivalent moment factor*" in the caption of Figure C-A-8.2
- "*Amplification factor*" on the ordinate axis of Figure C-A-8.2 and in the heading of Table C-A-8.1
- "*Coefficient assuming no lateral translation of the frame*" in Section 8.2.1 of the Appendix 8

The third definition is in direct contradiction to the contents of the Figure and of the Table (C_m is less than unity in most cases), and the meaning of the fourth definition is difficult to perceive.

3. Basic difference between the two C_m factors

Beam-columns have three ultimate limit states, namely: section yielding, in-plane instability and out-of-plane buckling. When the latter is take care by Eq. (7), Eq. (6) is responsible only for the first two.

For examining the section yielding, the bending moment is to be multiplied by the P–δ amplification factor B_1, the physical meaning of which is (Fig. 1)

$$B_1 = \frac{\text{max second-order moment}}{\text{max first-order moment}} = \frac{M^{II}_{max}}{M_2} \tag{9}$$

In this situation, B_1 should not be less than unity. Substitution in Eq. (4) leads to

$$C_{m,yi} = \frac{M^{II}_{max}}{M_2}\left(1 - \frac{P_u}{P_{e1}}\right) \tag{10}$$

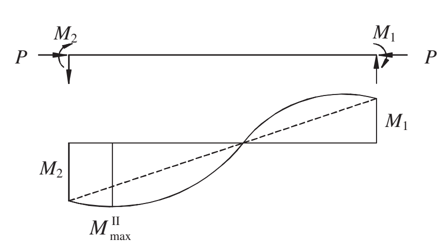

Fig. 1. Second-order moment.