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Efficient method for probabilistic finite element analysis with application to reinforced concrete slabs

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ABSTRACT

In this paper, probabilistic finite element analysis (FEA) is applied using the Monte Carlo simulation (MCS) and the multiplicative dimensional reduction method (M-DRM). M-DRM is proposed for stochastic FEA of large scale and/or complex problems, as it provides the probability distribution of the structural response, apart the statistical moments, and requires fairly small computational time. MCS and M-DRM results are compared, indicating that both are in a good agreement. In addition, sensitivity analysis is also performed using the M-DRM, which does not require any extra analytical effort. The probabilistic FEA is applied with the use of the ABAQUS software, where the development of the FEA model and the updating of each input random variable for the required simulations, are both implemented with the use of the Python programing language. Two previously tested reinforced concrete flat slabs, with and without shear reinforcement, are examined. The concrete damaged plasticity model is used for the modeling of the concrete, which is offered in ABAQUS. The results of the deterministic FEA simulation show reasonable response compared to the behavior of the test specimens in terms of ultimate load, deflection and cracking propagation. For the probabilistic analysis, only the material uncertainty is taken into account, in order to examine the accuracy and efficiency of the proposed M-DRM framework and the contribution of the material uncertainty to the output response. Finally, design codes (ACI 318-11 and EC2 2004) for punching shear and the critical shear crack theory (CSCT 2008, 2009) are examined, considering the same input uncertainties. Useful outcomes are presented indicating the predictive capability of the proposed probabilistic FEA for future studies.

1. Introduction

Finite element method (FEM) is the numerical approach which is used to solve approximately partial differential equations and finite element analysis (FEA) is the computational technique which is used in engineering in order to obtain approximate solutions of boundary value problems [1]. As a result, FEA is used to predict the structural response using numerical techniques to simulate the linear and/or nonlinear behavior of structural elements [2]. However, FEA predictions can be performed in a probabilistic sense due to unavoidable uncertainties in material, load parameters, modeling, etc. [3]. Therefore, FEA should be coupled with reliability analysis, often termed as finite element reliability analysis (FERA) [3] or stochastic finite element analysis (SFEA) [4].

In SFEA the input parameters are characterized as random variables and techniques such as the Monte Carlo simulation (MCS) can be used, in order to compute the statistical moments of structural response and the probability of structural response exceeding a safety threshold. In probabilistic FEA of large scale structures the following issues need to be addressed: (1) minimizing the number of FEA trials, especially when the deterministic analysis of the model is time consuming, (2) estimating accurately the probability distribution of the structural response, especially in FEA where the structural response function is in an implicit form, (3) connecting a general FEA software with uncertainty, especially when knowledge on advanced programing languages is required.

In order to apply probabilistic FEA, it is required to link a general purpose FEA program, e.g., ABAQUS, with an existing reliability platform, e.g., NESSUS [5] or ISIGHT [6], where more information regarding the connection between FEA software and structural reliability can be found in literature [7]. Another option is to use general-purpose and high-level programing languages. For example, Python development environment (PDE) is supported from ABAQUS graphical user interface (GUI). Thus, in the present work,

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Notation

A_s	cross-section area of reinforcement	w_j	Gauss quadrature weights
E_s	modulus of elasticity of reinforcement	x_j	Gauss quadrature coordinates for each input random variable
G_f	fracture energy	z_j	Gauss quadrature coordinates
$H[f]$	entropy of the response	α_i	fractional exponents of an ith fractional moment $(i = 1, 2, \ldots, m)$
$M_Y^{\alpha_i}$	fractional moment of the response	γ_c	density of concrete
S_i	primary sensitivity coefficient of each input random variable	γ_s	density of reinforcement
V_Y	variance of the response	θ_i	mean square of an ith cut function $(i = 1, 2, \ldots, n)$
Y	response	λ_i	Lagrange multipliers of an ith fractional moment $(i = 1, 2, \ldots, m)$
f'_c	compressive strength of concrete	μ	mean of each input random variable
f'_t	tensile strength of concrete	v	Poisson's ratio
f_y	yield strength of reinforcement	μ_Y	mean of the response
$f_Y(y)$	actual distribution of the response	μ_{2Y}	mean square of the response
$\hat{f}_Y(y)$	estimated distribution of the response	ρ_i	mean of an ith cut function $(i = 1, 2, \ldots, n)$
$h_i(x_j)$	response of the ith cut function when it is set at the jth Gauss quadrature point	σ	standard deviation of each input random variable
h_0	response when all input random variables are set equal to their mean values	σ_Y	standard deviation of the response
p_f	probability of failure		

Python code is developed in order to automate the required FEA trails, needed by the MCS and the proposed multiplicative dimensional reduction method (M-DRM).

Firstly, is shown how M-DRM provides the statistical moments, such as mean and variance, and the probability distribution of the structural response, requiring fairly small computational cost. The sensitivity analysis is also presented, where the sensitivity coefficients are obtained as a by-product of the M-DRM analysis. Then, the parameter updating idea in each simulation is adopted for the implementation of MCS in ABAQUS using Python programing language. M-DRM using the Gauss scheme also adopts the parameter updating idea in each trial and is automated in ABAQUS using Python programing. These two methods (M-DRM, MCS) are used for the probabilistic FEA of two interior reinforced concrete flat slabs. Prior to probabilistic analysis, deterministic nonlinear FEA is conducted using the concrete damaged plasticity model, in order to predict the structural behavior accurately. It is shown that M-DRM can overcome probabilistic FEA limitations efficiently, making it an easy to use technique. Then, is investigated how uncertainty, associated with the model's input parameters, impacts the structural response of the analyzed interior flat slabs. Comparison between the two slabs (shear unreinforced and shear reinforced) is performed based on the calculated load capacity of the slabs. Probabilistic analysis using design codes (ACI 318-11, EC2 2004) and a punching shear model (CSCT 2008, 2009) are critically compared to the probabilistic FEA results.

2. Multiplicative dimensional reduction method (M-DRM)

2.1. Background

In structural reliability analysis, the response can be modeled as a function of several input random variables [8]. For instance, punching shear strength is the output variable of interest when evaluating the capacity of a flat slab, which can be calculated as a function of input random variables such as the strength of concrete, the effective depth of slab, etc., as

$$Y = h(\mathbf{x}) \tag{1}$$

where Y is a scalar response and $\mathbf{x}$ is a vector of input random variables, i.e., $\mathbf{x} = x_1, x_2, \ldots, x_n$.

If we know the probability distribution of all variables $\mathbf{x}$, then we can calculate the probability of failure as

$$p_f = p(y_c - h(\mathbf{x}) \leqslant 0) \tag{2}$$

where p_f is the probability of failure and y_c is a critical threshold, where each response larger than this threshold leads to a structural failure. For simplicity, the probability of failure can be further described by the following integral [9]

$$p_f = \int_{\{g(\mathbf{x}) \leqslant 0\}} f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} \tag{3}$$

where $f_{\mathbf{x}}(\mathbf{x})$ is the joint Probability Density Function (PDF) of the previously defined vector $\mathbf{x}, \{g(\mathbf{x}) \leqslant 0\}$ represents the failure domain and $g(\mathbf{x}) = y_c - h(\mathbf{x})$ represents the performance function.

The above integral can be computed using [10]; (1) Direct integration, but the joint PDF is hardly available for real problems as it is defined implicitly in a finite element model; (2) Simulations, such as Monte Carlo simulation (MCS), but this method usually requires considerable computational time for a nonlinear finite element model; (3) Approximate methods, such as first order reliability method (FORM), but requires iterations which may give inaccurate solutions due to the nonlinearity of the limit state function of the finite element model.

Another option is to use the multiplicative dimensional reduction method (M-DRM), as it requires little computational cost and no iterations. M-DRM provides the probability distribution of response from which we can calculate the probability of exceedance, i.e., probability of failure, as shown in next sections. In literature [11–13], additive dimensional reduction method (A-DRM) has been used to approximate the response function in an additive form as

$$Y = h(\mathbf{x}) \approx \sum_{i=1}^{n} h_i(x_i) - (n-1)h_0 \tag{4}$$

A-DRM is not practical for the computation of fractional moments [14] while M-DRM has the benefit to simplify the evaluation of both integer and fractional moments of the response [15]. Using logarithmic transformation on Eq. (4), M-DRM approximates the response function in a multiplicative form as

$$Y = h(\mathbf{x}) \approx h_0^{(1-n)} \times \prod_{i=1}^{n} h_i(x_i) \tag{5}$$