Engineering Structures 103 (2015) 125–133

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Application of influence lines for the ultimate capacity of beams under moving loads

Graziano Fiorillo *, Michel Ghosn [1]

Department of Civil Engineering, The City College of New York/CUNY, 160 Convent Avenue, New York, NY 10031, United States

ARTICLE INFO

Article history:
Received 21 March 2015
Revised 11 August 2015
Accepted 2 September 2015
Available online 18 September 2015

Keywords:
Influence line
Failure mechanism
Nonlinear analysis
Ultimate capacity
Critical load location

ABSTRACT

This paper describes a procedure for estimating the nonlinear ultimate load carrying capacity of continuous beams using a combination of linear elastic influence lines for segments of the beam in appropriately selected damaged configurations. The method extends the use of influence lines beyond the elastic limits while preserving their inherent efficiency and accuracy. The approach is validated by comparing it to numerical results obtained through structural nonlinear analysis software and other analytical solutions. A main advantage of the method is its ability to determine the most critical locations for placing multiple point loads that cause the beam's plastic failure. This method may be of practical interest in structural and bridge engineering when numerous nonlinear analyses are required to estimate the ultimate capacity of structures that may be subject to moving loads.

1. Introduction

The use of nonlinear analysis models in structural engineering is becoming more common in professional practice that aims to obtain more accurate estimates of the ultimate load carrying capacities of new and existing structures. This trend is accelerating due to the widespread availability of computer programs that can efficiently and accurately obtain numerical solutions to complex structural systems [15]. Although any type of structure can be analyzed using proper three-dimensional (3D) nonlinear finite element methods, the simplicity of beam theory makes it an important tool for application in civil, structural, and mechanical engineering. In this context, when dealing with the nonlinear behavior of beam structures, it is necessary to include plasticity and nonlinear damage in the beam model [10]. The plastic behavior of beams is usually accounted for through two types of nonlinear modeling techniques. The first approach applies lumped plastic hinges at specific sections of the beams where the plastic behavior is assumed to be concentrated [11,2], while the other method adopts distributed plastic fibers along the members [14]. The mathematical formulation of the concentrated plastic hinge method is simpler than the distributed plasticity model which explains why the former has been more commonly applied in structural engineering practice [1,3].

While applications of nonlinear beam and frame analysis are widely available for different applications, its use for the nonlinear analysis of bridges subjected to moving traffic load is gaining popularity due to current attempts to obtain more accurate estimates of the load carrying capacity of bridge systems by considering their ultimate capacities, structural redundancy and robustness [6,7,13].

When performing the nonlinear analysis of bridges under the effect of moving loads, engineers have traditionally located the critical position of the moving loads based on a linear elastic analysis by often using classical influence lines [8,12,4]. This approach was found to be especially practical when solving problems that require large numbers of analyses such as when performing reliability analyses. The objective of this paper is to describe a computationally efficient procedure that gives an estimate of the beam's ultimate capacity using influence lines. The procedure is shown to improve our ability to determine the most critical locations where the loads should be placed when performing the nonlinear structural analysis reducing the number of load patterns that need to be investigated.

2. Review of influence lines

Certain types of structures, such as bridges, are loaded by both non-transient and transient loads (such as those representing vehicles). The transient nature of the loads implies that numerous load configurations are possible. Influence lines provide an important tool to identify the most critical loading configuration and loading

* Corresponding author. Tel.: +1 (212) 650 7000; fax: +1 (212) 650 6965.
E-mail addresses: gfiorillo@ccny.cuny.edu (G. Fiorillo), mghosn@ccny.cuny.edu (M. Ghosn).
[1] Tel.: +1 (212) 650 8002; fax: +1 (212) 650 6965.

http://dx.doi.org/10.1016/j.engstruct.2015.09.003
0141-0296/

points that will create the most severe effects on the various components of the structure. An influence line describes the variation of a load effect (such as the moment, shear force, reaction or deflection) at a specific point or component of a structure as a concentrated loading action (force, moment or temperature) moves over the structure. For example, Fig. 1 shows plots of typical influence lines for the shear force $\lambda_V(x)$ and moment $\lambda_M(x)$ at cross section "S" of a simply supported and a two-span continuous beam due to a unit force as it moves to various positions "x" along the length of the beams. The concept of the influence line can also be extended to two dimensions. In that case, the analysis is based on *influence surfaces* rather than influence lines [17]. Traditionally, influence lines have been derived using the Betti-Maxwell reciprocity theorem and Muller-Breslau's principle [5,17] although there are several other analytical and numerical approaches that can be used [9].

Influence lines are constructed for different sections and components to determine the most critical sections or components of a structure and identify the most critical loading position for each load effect. For example, Fig. 2 shows the influence lines for moment and shear of a three-span continuous beam with span lengths equal to 100 ft (30.0 m), 120 ft (37.0 m) and 100 ft (30.0 m). The top plot shows three moment influence line diagrams corresponding to three different sections along the continuous beam, specifically at 40 ft (12.0 m), 100 ft (30.0 m) and 160 ft (18.5 m) for a unit force. The bottom graph of Fig. 2 shows the shear influence lines for the same sections.

3. Classic failure mechanism formulation

Because the calculation of the influence line is based on the linear elastic behavior of the structure, it can only be directly used to identify the most critical load position which will cause the most critical component to reach its elastic limit. However, the same load position will not necessarily coincide with the one that will produce the most likely collapse mechanism. To demonstrate this concept let us consider the three-span continuous beam of Fig. 3. Assuming the nonlinear behavior is represented using the model of concentrated plasticity, the ultimate moment capacity of each beam section can be represented by $M_{u,i}$ which can be modeled by a rigid-plastic moment-rotation curve. The latter assumption does not invalidate the generalization of the approach to more complex nonlinear behavior models because collapse will only be a function of the ultimate moments $M_{u,i}$ at different critical sections i that have been plasticized. For the load pattern shown in Fig. 3, the failure mechanism requires the plasticization of the two sections S and B. The maximum Force F_u that produces the failure of the structure for this configuration can be calculated using the virtual work principle and it is equal to the expression in Eq. (1) if the values of $M_{u,S}$ and $M_{u,B}$ are the same.

$$F_u = M_u \cdot \frac{L + x_S}{(L - x_S) \cdot x_S} \tag{1}$$

where L is the length of the span under investigation and x_S is the distance of section S from the support A.

In practical applications, engineers are interested in identifying the positions of the load along the length of the beam that will cause collapse with the minimum load intensity. When the load is characterized by a single force, this problem is simple to solve. In fact, by taking the derivative of Eq. (1) with respect to x_S and setting it equal to zero, the solution of the equation for x_S gives the minimum value for F_u:

$$\frac{dF_u}{dx_S} = \frac{d}{dx_S}\left(M_u \cdot \frac{L + x_S}{(L - x_S) \cdot x_S}\right) = 0. \tag{2}$$

The solution of Eq. (2) gives $x_S = 0.414\ L$ which is consistent with the result of the linear elastic analysis that identifies the section at 0.414 L as the most critical for first failure in positive bending. If $M_{u,S}$ and $M_{u,B}$ are not equal, then the expression in Eq. (1) is no longer valid and F_u and x_S obtained above are no longer correct. The same is true when there are more than one applied force. For this reason, the virtual work approach is not recommended because it requires setting up a different ultimate load equation for each structural configuration and each loading configuration. This justifies the need for the approach proposed in this study based on an alternative method which extends the use of influence lines for evaluating the ultimate capacity of beams as described in the following section.

4. Proposed methodology

The example described in the previous section shows the impracticality of the virtual work method when the failure mechanism due to the applied load changes along the length of the beam. In general, a flexural beam under high vertical loads can fail in shear, bending or a combination of both. For the middle beam of a three-span continuous system, three plastic hinges are required to cause a bending moment mode of failure as shown schematically in Fig. 4. Also, the member would fail in shear if only two plastic shear hinges are formed as shown in Fig. 5. The two-hinge shear failure mechanism is valid whether the beam sections shear behavior is considered to have some level of ductility or is considered to be brittle. For sections with brittle behavior, the analysis procedure for the ultimate capacity is simpler than the case of beams with ductile behavior as it will be described later at the end of this section. Let us first discuss the case of ductile beams. Figs. 4 and 5 show the failure mechanisms and the influence lines associated with each section evaluated for the initial structural configuration without plastic hinges labeled as sections B, S and C. After the first plastic hinge is formed, a new structural configuration is defined by adding a release where the plastic hinge forms and the additional load is redistributed according to the new configuration. Therefore, any possible failure mechanism can be obtained by combining the influence lines based on the status of critical sections. For instance, each section may or may not have undergone local plasticization which is reflected by the presence of a hinge. Moment or shear failures at three critical locations of the beam are indicated with letters B, S, and C in Figs. 4 and 5. A digit is assigned to each section to indicate the status of the section, 0 indicates the undamaged section, number 1 and 2 indicate the section moment and shear plasticization respectively. All possible member failure possibilities are listed in Table 1. The first three columns of Table 1 shows the status of the section at location B, S, and C, while the fourth column describes the status of the member for each combination. These combinations represent a potential configuration of the member prior to failure. The total number of possible permutations is equal to 16 after removing the number of statically inadmissible cases. For example the combination [2, 2, 2] is not an admissible combination, because it represents the failure in shear at three sections B, S and C, while any of two shear plastic hinges in the system would have already caused failure of the member. It is proven in this study that the failure causing load of the beam for any loading position along the beam can be estimated once the influence lines of the structure for the 16 cases listed in Table 1 are established.

As shown in Figs. 4 and 5, a continuous beam subjected to a generic set of forces will need to develop at most three plastic hinges at points B, S, and C to produce a mechanism either for pure bending or a combined effect of moment and shear while just two hinges are sufficient to cause failure in pure shear. Let us indicate