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Inelastic mixed fiber beam finite element for steel cyclic behavior

I.A. Gkimousis, V.K. Koumousis *

Institute of Structural Analysis & Aseismic Research, National Technical University of Athens, Zografou Campus, 15780 Athens, Greece

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ABSTRACT

In this work a new hysteretic uniaxial steel model is determined to describe steel cyclic behavior, which is further implemented to derive a fiber beam–column element on the basis of Hellinger–Reissner principle. The proposed model maintains full memory of the loading path and evolves following a single nonlinear differential equation expressing the entire hysteresis. The element is capable of addressing the overshooting problem of the existing models which occurs during short reversals. The state determination of the proposed element is investigated numerically following the linearization of the derived equations. Numerical results are presented that validate the proposed approach and demonstrate its computational efficiency.

1. Introduction

The state of a deformable body subjected to body forces, tractions and kinematic boundary conditions is considered fully defined when the displacements, stresses and deformations are determined at any point of the body. In particular, for earthquake engineering and structural analysis of skeletal structures, beam elements usually based on the Euler–Bernoulli theory assumption that plane sections remain plane and perpendicular to the deformed axis, are typically considered. This facilitates considerably state determination of such elements.

Displacement based beam elements were initially used following the classical stiffness method in which displacements were the only considered independent field [1]. When cubic and linear shape functions are employed for the transverse and axial displacements respectively, the resulting displacement field leads to constant axial deformation and linear curvature, which however is not appropriate when plastic deformations occur. To address this deficiency a structural member should be discretized in more than one element at the expense of increasing computational cost. Also, equilibrium equations are only fully satisfied at element nodes, while within the element they are satisfied in weak form as they are not valid for all possible displacement fields that satisfy essential boundary conditions.

To resolve this problem, force based models were proposed that interpolate nodal forces within the element maintaining equilibrium. These models were implemented in the framework of the stiffness method of structural analysis and in that respect they are considered "mixed" as they use both force and displacement fields as independent ones. One of the first consistent and general force based beam model was proposed by Spacone et al. [2] and was later simplified numerically by Neuenhofer and Filippou [3]. Although the force based method proved very efficient and is currently widely used, there were some concerns about its variational consistency that were resolved by Hjelmstad and Taciroglu [4]. Moreover, the same authors in [5] showed that it is possible to provide non-variationally consistent force-based elements within the "nonlinear flexibility" framework by enforcing equilibrium directly. Also, various local and global solution strategies originate from the variational structure of the mixed beam elements as described in [6,7]. Thereafter, mixed methods seem to dominate the research field of nonlinear beam problems and corresponding numerical procedures as they proved more efficient following also the work of Hjelmstad and Taciroglu [8], Taylor et al. [9], Alemdar and White [10], Alsafadie et al. [11] and Correia et al. [12].

In this context elastoplastic material models were represented in classical form relying on the notions of yield surface, flow rule and hardening parameters, while their incorporation in the state determination process in linearized form gave rise to the return-mapping algorithm [13]. However, cyclic behavior can also be modeled using hysteretic evolution differential equations. Following this approach, Simeonov et al. [14] developed a force based element where material constitutive relations are considered in rate form and are solved simultaneously with the global differential equations of motion in state-space form. Also, Jafari et al. [15] extended

* Corresponding author. Tel.: +30 2107721657; fax: +30 2107721651.
E-mail addresses: cvgkim@central.ntua.gr (I.A. Gkimousis), vkoum@central.ntua.gr (V.K. Koumousis).

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this formulation in large displacement analysis following a displacement based formulation. In addition Triantafyllou and Koumousis [16,17] proposed a finite element procedure where material nonlinearity is treated constitutively at the element level through proper implementation of the Bouc–Wen hysteretic rule.

In the case of elastoplastic steel cyclic behavior various models have been proposed among which the widely used Menegotto–Pinto model [18] originating from the generalization of Ramberg–Osgood model [19]. Moreover, for efficient modeling of the inelastic buckling of reinforcing steel bars under cyclic behavior, the Monti–Nuti model [20] is commonly used as an enhancement of the Menegotto–Pinto model, which accounts for four different hardening rules. Furthermore, inelastic buckling of reinforcing bars has been investigated in several other studies, i.e. [21–24], while also modeling of high strength structural steel is addressed in Refs. [25,26].

Although the core of several steel models is the Menegotto–Pinto model, it is characterized by an overshooting of the reloading branch after short reversals [27]. This feature was considered of minor importance and originates from an effort to reduce computational cost by truncating model's memory. Although attempts were made to tackle this aspect by distinguishing the reversals as major and minor [28] or complete and incomplete [29] the problem can be also addressed from a different perspective.

In this work a new small displacement fiber beam–column element is proposed, which incorporates a uniaxial steel model for cyclic loading that resolves the overshooting behavior after short reversals, being on the same time computationally efficient. To accomplish this, a hysteretic model is developed that maintains full memory of the loading path and evolves following a single differential equation expressing the entire hysteresis. This constitutive behavior is implemented in the general framework of the two-field Hellinger–Reissner formulation [30] resulting in a well-established state determination algorithm. This is further investigated numerically following the linearization of the equilibrium and compatibility element equations.

The rest of the paper is organized as follows: In the first section the basic structure of the proposed model is derived from classical plasticity considerations incorporating kinematic hardening. Then, the modifications for modeling steel cyclic behavior are implemented and the comparison with the Menegotto–Pinto model is performed underlining the features of the proposed formulation. Cross-sectional constitutive equations are derived in the sequel from fiber integration and the two-field Hellinger–Reissner principle is used to derive the element equilibrium and compatibility equations. The standard linearization method is implemented for solving the element equations and the respective state determination process is discussed. Finally, three numerical examples are presented that verify the proposed beam element and demonstrate the performance of the hysteretic constitutive relations embedded into a two-field variational formulation for the inelastic analysis of steel frames.

2. Inelastic Euler–Bernoulli beam theory

2.1. Fiber plasticity

A hysteretic model incorporates the entire inelastic loading path of a deformable body, namely elastic loading, yielding, hardening and unloading in a single nonlinear differential equation that embodies both the yield surface and hardening rule. Mathematically this addresses the entire evolution process without the need of incremental considerations [31]. In this context Sivaselvan and Reinhorn [32] based on Bouc–Wen model [33] proposed a hysteretic model in stress resultant terms derived explicitly from classical plasticity theory.

In this work the stress–strain constitutive law of an elastoplastic fiber-rod subjected to uniaxial tension is determined according to the strain decomposition rule in rate form as [34]:

$$\dot{\sigma} = E(\dot{\varepsilon} - \dot{\varepsilon}^p) \tag{1}$$

where $\dot{\sigma}$ is the rate of normal stress, $\dot{\varepsilon}$ is the rate of total strain and $\dot{\varepsilon}^p$ is the rate of plastic strain. This relation indicates that stresses evolve proportionally to the evolution of the elastic strains. For the same fiber a yield function with linear kinematic hardening and the same initial yield stress in tension and compression $\left(\sigma_{y0}^{+} = \sigma_{y0}^{-} = \sigma_{y0}\right)$ is expressed in the following form:

$$\Phi(\sigma, b, \sigma_{y0}) = |\sigma - b| - \sigma_{y0} \leqslant 0 \tag{2}$$

with b being the back stress. Plastic strain rate $\dot{\varepsilon}^p$ is determined according to the flow rule:

$$\dot{\varepsilon}^p = \dot{\lambda} \cdot \frac{\partial \Phi}{\partial \sigma} = \dot{\lambda} \cdot sgn(\sigma - b) \tag{3}$$

where $\dot{\lambda} \geqslant 0$ is the plastic multiplier which is actually the magnitude of the strain rate, with the signum function defining its sense. Axial stress and plastic multiplier are restricted by unilateral constrains representing restrictions that signify whether the material has yielded or not, resulting from Karush–Kuhn–Tucker (KKT) optimality conditions. These are expressed in the following form:

$$\dot{\lambda} \cdot \Phi(\sigma, b, \sigma_{y0}) = 0 \tag{4}$$

During plastic response $(\dot{\lambda} > 0,\ \Phi(\sigma, b, \sigma_{y0}) = 0)$ the consistency condition is derived by differentiating Eq. (4):

$$\dot{\lambda} \cdot \dot{\Phi}(\sigma, b, \sigma_{y0}) = 0 \ \Rightarrow\ \dot{\Phi}(\sigma, b, \sigma_{y0}) = 0 \ \Rightarrow\ \dot{\sigma} = \dot{b} \tag{5}$$

Also, when the fiber deforms elastically $(\dot{\lambda} = 0,\ \Phi(\sigma, b, \sigma_{y0}) < 0)$ the sign of the rate of the yield function indicates loading or unloading with $\dot{\Phi} > 0$ and $\dot{\Phi} < 0$ respectively.

Taking into account Eq. (5) meaning that the rate of the back stress during plastic loading is equal to the rate of the axial stress, the following relation is considered:

$$\dot{b} = H \cdot \dot{\varepsilon}^p = H \cdot \dot{\lambda} \cdot sgn(\sigma - b) \tag{6}$$

where H is the hardening ratio, i.e. the slope of the stress–strain curve in plastic strain terms (Fig. 1). Substituting Eq. (1), (3) and (6) in Eq. (5) the following relation is obtained:

$$\dot{\lambda} = sgn(\sigma - b) \cdot \frac{E}{E + H} \cdot \dot{\varepsilon} \tag{7}$$

Furthermore, introducing $a = E_t/E$ as the ratio of the post-yield tangent modulus over the elastic modulus, the relation between the hardening ratio H and tangent modulus E_t is established as:

$$H = \frac{E_t}{1 - E_t/E} = \frac{a}{1 - a} \cdot E \tag{8}$$

Then Eq. (3), using relations (7) and (8) obtains the following simple form:

$$\dot{\varepsilon}^p = (1 - a) \cdot \dot{\varepsilon} \tag{9}$$

Eq. (9) holds only for the plastic deformation phase. During elastic loading or unloading plastic strain is not induced as $\dot{\lambda} = 0$. This is controlled by the compact form of KKT conditions (4). To include all loading phases in a single relation, Eq. (9) is extended by considering two Heaviside type functions acting as switches as follows:

$$\dot{\varepsilon}^p = H_1 \cdot H_2 \cdot (1 - a) \cdot \dot{\varepsilon} \tag{10}$$

where H_1 controls yielding and H_2 controls loading/unloading states. Consequently the following distinct phases can be described in a single equation: