Engineering Structures 108 (2016) 47–52

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

In-plane behaviour of web-tapered beams

N.S. Trahair *, P. Ansourian

School of Civil Engineering, The University of Sydney, NSW 2006, Australia

ARTICLE INFO

Article history:
Received 7 October 2015
Accepted 11 November 2015
Available online 9 December 2015

Keywords:
Bending
Deflections
Elasticity
Force
I-beam
Normal stress
Shear
Shear stress
Tapered web
Edge

ABSTRACT

Shear stress distributions in tapered web I-beams are incorrectly predicted by the conventional beam analysis method used for uniform beams. More accurate predictions are obtained by adopting the finding for wedges that the normal stress trajectories are radial instead of parallel.

The shear stress distributions in web-tapered I-beams are influenced by the vertical components of the inclined flange forces (which are zero in uniform beams), as well as by the normal stress gradients in the flanges. The net web shear equal to the difference between the external shear and the vertical components of the inclined flange forces is resisted by the resultant of the vertical components of the normal stresses and the circumferential shear stresses.

The circumferential shear stresses $\tau_{r\theta}$ have linear components due to axial force and parabolic components due to moment and shear. The magnitudes of these stresses are controlled by the normal stress gradients at the flange-web junctions and by the requirement that the web shear resistance must equal the net web shear force.

1. Introduction

The in-plane behaviour of tapered I-beams (Fig. 1) is rarely treated in textbooks, but designers commonly assume that they behave in the same way as uniform beams. While this may be satisfactory for the bending deflections and the normal stresses, it may lead to incorrect shear stress distributions.

In uniform I-section beams, the normal stresses due to moments and axial forces are parallel to the centroidal axis, while the shear forces are resisted solely by shear stresses in the web [1]. However, in web-tapered beams, the flanges are inclined to the centroidal axis, as are the flange normal stresses, and so the stress trajectories are inclined to the centroidal axis. In addition, the inclined flange forces have components transverse to the centroidal axis, which may participate in resisting the shear forces.

In this paper, the distributions of the normal and shear stresses in web-tapered I-beams are investigated. First, the known stress distributions in tapered plate wedges (Fig. 2) are reviewed, because of the similarity of tapered wedges to the webs of tapered I-beams. These suggest that the normal stresses in web-tapered I-beams might be assumed to be radial. This assumption is then used to analyse the distributions of normal and shear stresses. Finally, the predicted stress distributions are compared with those obtained from a more rigorous finite element program [2] which incorporates the two-dimensional membrane behaviour of the flange and web plates of which the tapered beams are composed.

2. Plate wedges

The stresses in wedges (Fig. 2) have been reported in [3]. For a wedges of included angle 2α with an end compression load $-N$ acting along the axis, the stress distribution at a distance r from the apex of the wedge is purely radial (Fig. 3a), with zero circumferential normal stresses σ_θ and shear stresses $\tau_{r\theta}$. The radial stresses

$$\sigma_r = -\frac{2N\cos\theta}{r(2\alpha + \sin 2\alpha)} \quad (1)$$

vary with the angle θ from the axis, but are nearly uniform for wedges with small taper angles α. The stress trajectories are clearly radial along the lines θ = constant, which are also principal stress contours.

For wedges with end loads V acting transverse to the axis, the stress distribution is again radial (Fig. 3b), with zero circumferential normal stresses σ_θ and shear stresses $\tau_{r\theta}$ and

$$\sigma_r = -\frac{2V\sin\theta}{r(2\alpha - \sin 2\alpha)} \quad (2)$$

* Corresponding author. Tel.: +61 2 9451 4874.
E-mail address: N.Trahair@civil.usyd.edu.au (N.S. Trahair).

http://dx.doi.org/10.1016/j.engstruct.2015.11.010
0141-0296/

Nomenclature

A	area of cross-section	V	external shear force
$b_{f,w}$	flange width and web depth	V_f	sum of vertical components of flange forces
b_{w0}	flange width at $z = 0$	v_P, w_P	displacements of $P(z, y)$
$C_{m,n}$	constants of integration	x, y	principal axes
E	Young's modulus of elasticity	z	distance along member
I_x	in-plane second moment of area	α	flange inclination
k	see Eq. (B.3)	ε_r	strain
L	length	θ	angle from z axis
M	external moment	σ_r	normal stress in r direction
M_c	mid-span moment	σ_z	normal stress in z direction
M_r	bending moment	$\tau_{r\theta}$	circumferential shear stress
N	external axial force	$\tau_{r\theta\alpha}$	value of $\tau_{r\theta}$ at bottom flange
r	distance of point $P(z, y)$ from flange intersection	τ_{0m}, τ_{0n}	see Eq. (B.11)
r_0	distance to beam left hand end	$\tau_{\alpha m}$	see Eq. (B.11)
$t_{f,w}$	flange and web thicknesses		
v, w	displacements in y, z directions		

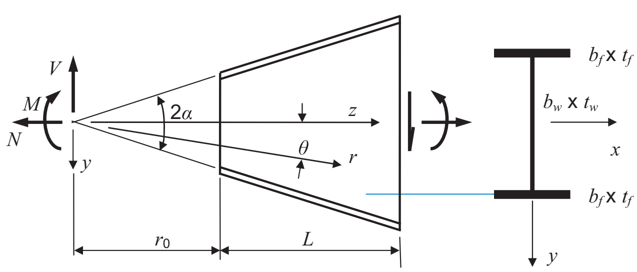

Fig. 1. Web-tapered I-beam.

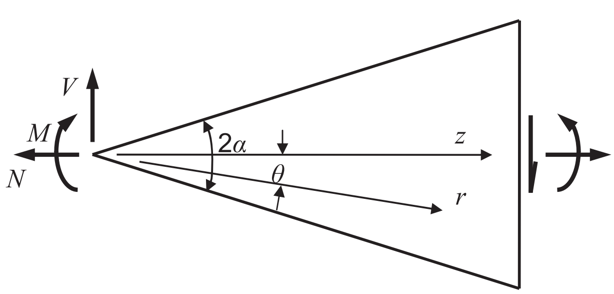

Fig. 2. Wedge.

The variation of these stresses with the angle θ is nearly linear for small taper angles α. Again, the stress trajectories are radial along the lines θ = constant, which are also principal stress contours.

For these wedges, the end load V causes bending moments and shear forces which vary along the axis. The variation of the horizontal components of σ_r in wedges with small tapers is close to the linear variation of σ_z for moment predicted by conventional beam analysis (CBA). However, the variation of the vertical components is very different to that of CBA, for which the distribution of the shear stress τ_{yz} is parabolic with zero stresses at the top and bottom edges, and 1.5 times the average at the axis. For the wedge, the shear stress τ_{yz} variation is also parabolic, but with zero stress at the axis and 3 times the average at the top and bottom edges. The sums of the horizontal and vertical components of σ_r are equal to the moment and shear effects of the applied load.

For wedges with end moments M, the circumferential normal stresses σ_θ are again zero, but the radial stresses are accompanied by shear stresses (Fig. 3c). The radial stresses are

$$\sigma_r = -\frac{4M\sin 2\theta}{2r^2(\sin 2\alpha - 2\alpha\cos 2\alpha)} \tag{3}$$

and the shear stresses are

$$\tau_{r\theta} = \frac{2M(\cos 2\theta - \cos 2\alpha)}{2r^2(\sin 2\alpha - 2\alpha\cos 2\alpha)} \tag{4}$$

The variation of the horizontal components σ_z in wedges with small tapers is close to the linear variation of CBA for moment. The vertical components of the radial stresses vary from maxima at the edges to zero at the axis, and are so balanced by the vertical components of the shear stresses that there is no vertical shear resultant. Because of the presence of shear stresses, the principal stress trajectories are not quite radial.

It can be concluded that the normal stress distributions in small taper wedges are radial, and are closely approximated by CBA. However, the shear stress distributions are quite different to those predicted by CBA.

3. Web-tapered beams

3.1. Beams

A simply supported web-tapered beam with equal uniform flanges is shown in Fig. 1. The beam length is L = 152 mm. The cross-section dimensions are shown in Table 1.

3.2. Methods of analysis

3.2.1. Tapered beam analysis (TBA)

The methods of linear elastic analysis (CBA) of uniform beams are well documented [1]. In summary, plane sections are assumed to remain plane and shear strains are neglected in the analysis of the bending deflections, and stress concentrations at applied loads or reactions are ignored. The section properties of area A and second moment of area I_x are used to determine the longitudinal normal stresses σ caused by axial force N and bending moment M_r, which are determined from the applied loads and moments either by equilibrium for determinate beams or by analysis of the bending deflections v for indeterminate beams. In Appendix A, CBA analysis of bending and compression is adapted to web-tapered I-beams by assuming that the normal stress trajectory inclinations