Engineering Structures 108 (2016) 53–58

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

In-plane behaviour of mono-symmetric tapered beams

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ARTICLE INFO

Article history:
Received 2 November 2015
Revised 14 November 2015
Accepted 14 November 2015
Available online 9 December 2015

Keywords:
Bending
Deflections
Elasticity
Force
I-beam
Mono-symmetry
Normal stress
Shear
Shear stress
Tapered flange
Edge

ABSTRACT

Shear stress distributions in mono-symmetric tapered I-beams are incorrectly predicted by the conventional beam analysis method used for uniform beams. More accurate predictions are obtained by assuming that the normal stress trajectories vary linearly between plate edges, instead of parallel to the centroidal axis.

Transverse shear stresses at the flange-web junctions of mono-symmetric tapered I-beams of constant depth are induced by gradients of the forces in the tapered flanges. The transverse shear stress distributions caused by axial force, moment, and shear force are constant, linear, and parabolic, respectively.

Axial force induces non-zero principal axis shear stresses, while shear force induces non-zero normal stresses acting parallel to the centroidal axis.

1. Introduction

While the in-plane behaviour of doubly symmetric tapered I-beams has recently been analysed [1], mono-symmetric tapered beams (Fig. 1) are rarely if ever treated in textbooks. Designers commonly assume that tapered beams behave in the same way as uniform beams, and while this is satisfactory for the bending deflections and the normal stresses, it may lead to incorrect shear stress distributions.

In uniform mono-symmetric I-beams, the normal stresses due to moments and axial forces are parallel to the centroidal axis, while the shear forces are resisted solely by shear stresses in the web [2]. However, in mono-symmetric tapered I-beams, the flanges are inclined to the centroidal axis, as are the flange normal stresses, and so the stress trajectories are inclined to the centroidal axis. In addition, the inclined flange forces have components transverse to the centroidal axis, which participate in resisting the shear forces.

In this paper, the distributions of the normal and shear stresses in mono-symmetric tapered I-beams are investigated. First, the stress distributions in mono-symmetric thickness tapered plates (without flanges) (Fig. 2) are analysed, because of their similarity to tapered wedges (without flanges), whose known behaviour [3] suggested [1] that the normal stresses in web-tapered I-beams act along lines whose inclinations vary linearly between those of the plate edges. This suggestion is then used to analyse the distributions of normal and shear stresses in mono-symmetric tapered I-beams (Fig. 1). Finally, the predicted stress distributions are compared with those obtained from a more rigorous finite element computer program [4] which incorporates the two-dimensional membrane behaviour of the flange and web plates of which the tapered beams are composed.

This paper has relevance to the lateral buckling of mono-symmetric tapered beams [5–7].

2. Tapered thickness plates

2.1. Plates

A tapered rectangular plate $d \times L \times t$ is shown in Fig. 2. The thickness t is tapered linearly along the length and down the depth. The thickness at the top LH and bottom RH corners is t_t, and at the bottom LH and top RH corners is t_b. The tapered thickness causes the plate to be mono-symmetric in cross-section and tapered along its length. The centroidal axis CC is inclined to the plate mid-line.

The conventional beam analysis (CBA) of uniform beams under axial compression and bending is adapted for tapered

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http://dx.doi.org/10.1016/j.engstruct.2015.11.011
0141-0296/

Notation

A	area of cross section	v, w	displacements in Y, Z directions
$b_{b,t}$	widths of bottom and top flanges	V	shear force
b_w	web depth	v_P, w	displacements of P(Z, Y)
d	plate depth	x, y	principal axes
E	Young's modulus of elasticity	Y, Z	vertical and horizontal axes
I_x	in-plane second moment of area	Y_c	distance to centroid
L	length	z	distance along centroidal axis
M	moment	α_t	taper angle of the top flange
N	axial force	ε_Z	strain
N_f	flange force	θ	angle between Z and z axes
$t_{b,t}$	bottom and top flange thicknesses	σ_Z, σ_z	normal stresses in Z, z directions
$t_{f,w}$	flange and web thicknesses	τ_{ZY}, τ_{zy}	shear stresses in Y, y directions

mono-symmetric plates in Appendix A by assuming that the normal stresses act along lines parallel to the longitudinal edges of the plates.

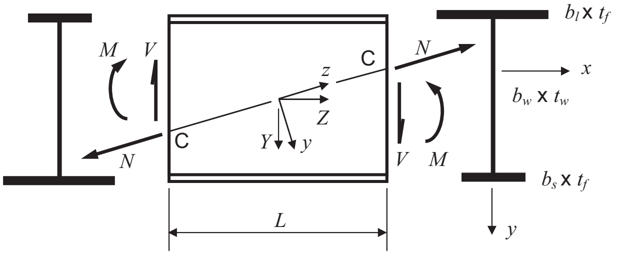
2.2. Uniform compression

Uniform compression of the tapered plate induces uniform longitudinal normal stresses σ_Z and transverse shear stresses τ_{ZY} which have a parabolic variation down the plate at mid-span from zero at the edges to a maximum at mid-depth, as shown in Fig. 3a.

These stresses may be converted to normal stresses σ_z parallel to the centroidal z axis and shear stresses τ_{zy} parallel to the principal y axis by assuming that the normal stresses σ_Y are zero, whence [3]

$$\begin{aligned} \sigma_z &= \sigma_Z \cos^2\theta + 2\tau_{ZY}\sin\theta\cos\theta \\ \tau_{zy} &= \tau_{ZY}(\cos^2\theta - \sin^2\theta) - \sigma_Z\sin\theta\cos\theta \end{aligned} \tag{1}$$

in which θ is the angle between the Z and the z axes. The principal axis stresses σ_z and τ_{zy} are also shown in Fig. 3a. The resultant of the normal stresses σ_z is equal to the applied compression force, while the non-zero shear stresses τ_{zy} have a zero stress resultant.



(a) Section 1 (b) Beam and Loading (c) Section 2

Fig. 1. Mono-symmetric tapered I-beam.

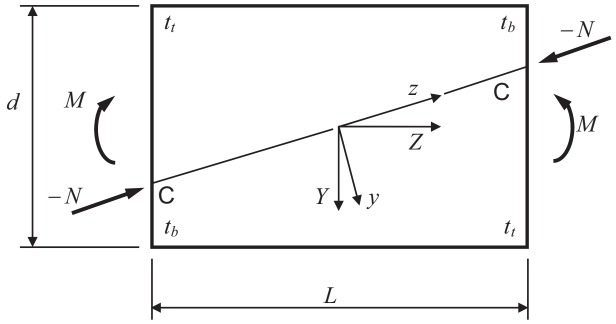


Fig. 2. Tapered mono-symmetric plate.

2.3. Uniform bending

Uniform bending induces longitudinal normal stresses σ_Z and shear stresses τ_{ZY}. At mid-span, the normal stresses vary linearly down the depth, and the self-equilibrating shear stresses have a cubic variation, as shown in Fig. 3b. The corresponding principal axis stresses σ_z and τ_{zy} are also shown in Fig. 3b. The resultant of the normal stresses σ_z is equal to the applied moment, while the non-zero shear stresses τ_{zy} have a zero stress resultant.

3. Mono-symmetric tapered I-beams

3.1. Beams

A mono-symmetric I-beam of constant depth and reversed tapered flanges is shown in Fig. 1. Its centroidal axis CC is inclined. The beam length is L = 152 mm. The cross-section dimensions are shown in Table 1.

3.2. Methods of analysis

3.2.1. Tapered beam analysis (TBA)

The methods of linear elastic analysis (CBA) of uniform beams are well documented [2]. In summary, plane sections are assumed to remain plane, shear strains are neglected in the analysis of the bending deflections, and stress concentrations at applied loads or reactions are ignored. The section properties of area A and second moment of area I_x are used to determine the longitudinal normal stresses σ caused by axial force N and bending moment M, which are determined from the applied loads and moments either by equilibrium for determinate beams or by analysis of the bending deflections v for indeterminate beams. In Appendix B, CBA analysis of bending and compression is adapted for mono-symmetric tapered I-beams by assuming that the normal stress trajectory inclinations vary linearly between those of the edges of the plates of which the beam is composed. This adapted analysis using inclined stress trajectories is referred to in this paper as tapered beam analysis (TBA).

3.2.2. Finite element analysis (FEA)

For this paper, the bending and compression of tapered I- beams has been analysed (FEA) by using the finite element computer program STRAND7 [4]. This software contains a variety of plate/shell elements for the analysis of plane stress/strain/axisymmetric and general shell structures. In the context of this