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Modeling of the strut-and-tie parameters of deep beams for shear strength prediction

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A R T I C L E I N F O

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A B S T R A C T

A generally accepted rational model to predict the shear strength of a structural member needs to satisfy Navier's three principles (force equilibrium, strain compatibility, and constitutive equations). However, a shear model satisfying these three principles usually requires rigorous computational effort. This study reveals that, if involved structural parameters are properly considered, a simple strut-and-tie model that merely satisfies force equilibrium can give similar accuracy compared to the sophisticated strain-compatible model. This finding was verified against 118 deep beam specimens tested in the laboratory. The important structural parameters identified are the definition of a shear element that is consistent with force discontinuity, the consideration of elastic behavior in estimating the width of a strut, the dimensions of a nodal zone influenced by a loading plate, and the proper selection of the probable failure modes.

1. Introduction

Since its introduction by Schlaich et al. [1], the strut-and-tie method (STM), which is based on the lower bound theory of plasticity, has served as a design and analytical tool for structural members, especially those with complicated flows of forces or deep members. The STM has been widely used by both engineers and researchers because the method provides a clear load path and is simple in terms of both the solution algorithm and the equations involved. Moreover, a computer program to assist engineers to analyze and design using STM was developed by Tjhin and Kuchma [2]. The main implication of the wide acceptance of the STM is its adoption into the building codes of many countries, such as Canada [3], the European Union [4], New Zealand [5], and the United States [6].

By contrast, several analytical models have also been derived. Zhang and Tan [7] proposed a strut-and-tie model that was based on the Mohr Coulomb's failure criterion. Several other researchers developed strut-and-tie models which satisfied Navier's three principles, such as the compatibility-based strut-and-tie that uses the secant stiffness formulation [8] and the softened strut-and-tie model [9,10]. For deep beam specimens loaded through a column stub, the softened strut-and-tie model was further adjusted to include the effect of boundary condition due to presence of a column stub [11]. These analytical models allow engineers to produce one unique solution that satisfies not only force equilibrium but also strain compatibility and a stress–strain relationship for cracked reinforced concrete. Some other models are based on a mechanical approach, such as the two-parameter kinematic theory [12], which was derived satisfying the kinematic of deep beam's deformation. These available models provided reasonable accuracy to the existing database, but they required rigorous computational effort.

It is commonly assumed that sophisticated solution algorithms produce better accuracy when predicting the shear capacity of deep beam specimens compared to the simple ACI 318 strut-and-tie equations. This assumption may be a misconception because not all of the available algorithms use the same macro model. In this case, the macro model corresponds to the idealized visualization of the load path and the geometry of the struts and ties that reflect the major parameters influencing the structural behavior of a deep beam.

This study begins with the strut-and-tie model (STM) provision described in ACI 318 [6] in which a direct force transfer mechanism is assumed. The analysis result indicates that the simple ACI 318 STM provides a too conservative shear strength prediction when gauged against a group of deep beam databases. A further investigation of this over conservative prediction suggests that it does not come from the simplicity of the solution algorithm, but rather from

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the improper modeling of the structural parameters inherent within the macro model. Therefore, the original macro model of ACI 318 is modified to include a structural behavior that represents the testing conditions and behavior of the deep beam specimens tested in the laboratory as well as the proper justification of the most probable failure modes. The results of the final modification of the model indicate that when all of these parameters are properly considered, the simple ACI 318 strut-and-tie equations provide a similar accuracy compared to the softened strut-and-tie model (SST). Meanwhile, further discussions related to the differences between ACI 318 strut-and-tie and SST model can be seen elsewhere [13].

2. Strut-and-tie model based on ACI 318

ACI 318 STM is presented in a simple form, as shown in Fig. 1. Once all of the beam dimensions, reinforcement detailing, material properties, and testing parameters are known, one may develop any macro model of a strut-and-tie that satisfies force equilibrium. As given by Eq. (1), the calculated shear strength of a deep beam (V_n) is determined as being the smallest of the following: the strengths of the diagonal strut at the top (V_1) and at the bottom (V_2), the strengths of the nodal zone at the top (V_3) and at the bottom (V_4), and the yielding of tension ties (V_5):

$$V_n = \min\{V_1, V_2, V_3, V_4, V_5\} \quad (1)$$

The strengths of the struts are empirically determined using strut efficiency factor 0.85 β_s, depending on the amount of the vertical and horizontal shear reinforcements crossing the strut [6]. The strengths of the nodal zone at the upper part (V_3) and the lower part (V_4) are determined using the β_n factor for the CCC (resists three compressive forces) and CCT (resists two compressive forces and one tensile force) nodes, respectively. Meanwhile, the strength of the tension tie (V_5) is taken as the yielding strength of the flexural reinforcement.

2.1. Macro model

Although a STM provides a clear force transfer mechanism of a modeled structure or a region, it does not rigidly specify how the load is transferred from the loading point (actuator) to the support. One may use a direct STM in which the load is transferred directly from the loading plate to the reaction plate or other truss models to consider the additional load paths due to the presence of vertical stirrups. The simplest STM, also adopted in this paper, uses a direct transfer mechanism in which the force from the loading actuator is directly transferred to the support reaction (Fig. 1). Brown and Bayrak [14] also concluded that the direct transfer mechanism might be considered as an appropriate mechanism, especially for deep beams with a shear span to depth ratio that is less than 2.

After determining the load path, one must define the macro model of the strut-and-tie, which includes the determination of the width of the horizontal strut w_s, the width of the horizontal tie w_t, and the inclination angle of the diagonal strut θ. The aforementioned width of the horizontal strut represents the depth of the compression zone at the constant bending moment region. According to Tjhin and Kuchma [15], the depth of the compression zone is taken as the plastic compression zone presented in Eq. (2):

$$w_s = \frac{A_s f_y}{0.85 \beta_s f'_c b} \quad (2)$$

where $A_s f_y$ is the yield strength of the main flexural reinforcement, β_s is a factor used to account for the effect of cracking on the effective compressive strength of a concrete strut given by ACI 318-14, f'_c is the compressive strength of concrete, and b is the width of the beam. In addition, the width of the horizontal tie is calculated by following the recommendation in ACI 318-14, as presented in Eq. (3):

$$w_t = \frac{A_s f_y}{0.85 \beta_n f'_c b} \quad (3)$$

where β_n in this case is the factor used to account for the effect of the anchorage ties on the effective compressive strength of a nodal zone taken at the lower part (CCT node).

The inclination angle relative to the horizontal axis θ of this concrete strut is given by Eq. (4):

$$\theta = \tan^{-1}\left(\frac{jd}{a}\right) = \tan^{-1}\left(\frac{d - w_s/2}{\ell_b/2 + a' + a_p/2}\right) \quad (4)$$

where jd is the force lever arm, a is the shear span, d is the effective depth of the beam, ℓ_b is the width of the bearing plate, a_p is the width of the loading plate, and a' is the clear shear span.Finally, after these three parameters are determined, the geometry of the nodal zone and the strut area at the top and bottom parts of the diagonal strut can be determined, as illustrated in Fig. 2(a). The nodal zone areas ($A_{cn,top}$; $A_{cn,bot}$), which are the same as the strut areas ($A_{cs,top}$; $A_{cs,bot}$), are defined such that they are perpendicular to the strut:

$$A_{cs,top} = A_{cn,top} = (w_s \cos\theta + a_p \sin\theta)b \quad (5)$$

$$A_{cs,bot} = A_{cn,bot} = (w_t \cos\theta + \ell_b \sin\theta)b \quad (6)$$

2.2. Verification of ACI 318 STM

To verify the accuracy of the STM of ACI 318 (Analysis 1), a database of the deep beam specimens that failed in shear was collected from the literature [16–21] and is presented in Table 1. The specimens used in this study were collected from the available literature, ensuring that the complete information of the test setup was provided. The database covers a wide range of concrete compressive strengths f'_c along with different layouts of reinforcements.

The shear strength ratios (V_{test}/V_{calc}) are plotted in Fig. 2(b), with each specimen represented by a number to indicate the predicted failure mode. The STM defined using the ACI 318 parameters predicts that the majority of specimens would fail in the upper part of the strut (failure mode 1). The STM also provides a very conservative and scattered strength prediction, as indicated by its average value of 1.54, and a coefficient of variation (COV) of 0.41. Fig. 2(b) also shows that the strength ratios for Kong et al.'s specimens are rather poor compared to others. This finding is because majority of Kong et al.'s specimens were detailed with small amount of longitudinal reinforcement ratio ($\rho \leqslant 0.8\%$). A further look into this finding suggests that the accuracy of the current ACI 318 STM is not so reliable for specimens with relatively low ratio of longitudinal reinforcement (Fig. 3). Therefore, the authors argue that in order to improve the accuracy of the current ACI 318 STM (Analysis 1), the macro model should reflect the structural behavior of deep beam specimens tested in the laboratory.

In the following section, a series of parametric analyses, with each analysis representing a macro model closer to the structural behavior of deep beam specimens, is performed. These analyses include the effect of force discontinuity on the appropriate determination of the shear element (Analysis 2), the selection of the compression zone to represent the width of the strut (Analysis 3), the influence of the steel loading plate on the dimension of the nodal zone (Analysis 4), and the selection of the probable failure mode (Analysis 5). In addition, using the same macro model as that used in the latest analysis (Analysis 5), this research also verifies the