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Dynamic analysis of a beam on block-and-tackle suspension system: A continuum approach

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ABSTRACT

In this paper the dynamic analysis of a beam on a block-and-tackle suspension system is accomplished using a continuum approach. The modal shape functions and the natural frequencies of the structure are derived in a dimensionless form for both slacked and stressed cables. A procedure is developed to handle the nonlinearity originated from the consecutive slacking and stressing of the suspension cable. Vibration analysis of the bilinear, multi-degree-of-freedom structure is accomplished for a vortex-shedding generated lift force and for a continuous pedestrian flow.

1. Introduction

Simple suspension bridges were already used more than 1000 years ago. The oldest known structure is from the 7th century, constructed by the *Maya* civilization at *Yaxchilan* [1]. Sketches of the first suspension bridge that resembles modern suspension and cable-stayed bridges appeared in *Fausto Veranzio*'s masterwork [2] in the late 15th century. These type of structures are composed of compressed pillars, a bridge deck, and cables. The main idea behind a suspension bridge is that there are (usually) two main cables that hang between the pillars and are anchored to the ground at both ends, while (vertical or inclined) suspenders connect the deck to the main cables. The cable-stayed bridges, on the other hand, have one or more pillars that are the main load bearing structures and inclined suspension cables transmit forces from the deck to the pillars. There are numerous variations of these kind of structures, see for example the comprehensive work of *Kawada* [3]. The length of suspension bridges varies from small footbridges, like the *Boston Public Garden Footbridge*, to the *Akashi Kaikyo Bridge*, whose central span is almost 2000 m long.

Longer and more slender bridges have appeared as material properties, design methods and building techniques have significantly improved. There has also been a strong community demand for more interesting structures, which are more aesthetic and appealing to the public. However, slender structures tend to be more sensitive to dynamic forces induced by wind loads [4,5] or traffic flow [6], for instance, resulting in vibrations of the bridge deck. These vibrations can attain high magnitude in some cases, especially when the vortex-shedding frequency of the wind or the pace of the traffic approaches one of the natural frequencies of the bridge. A well-known example of failure caused by mechanical and aerodynamic effects is the collapse of the *Tacoma Narrows Bridge* [7]. Pedestrian-induced vibrations of slender footbridges have also been analyzed by numerous authors. For a literature review of lateral vibrations see [6], while for vertical vibrations see for example [8] and the references therein. The most well-known example for dense pedestrian flow induced resonance of lateral vibration mode is the *London Millennium Footbridge* [9]. These examples have revealed that a proper dynamic analysis is necessary for slender bridges subjected to wind and traffic loads.

The application of some kind of suspension system for footbridge constructions is quite general. The disadvantage of cable suspension systems is that some cables can be highly overstressed while others can be slacked. High tension in cables is not desirable because it may lead to failure, but slacking of cables is also disadvantageous. Because cables do not have any resistance to compression, the dynamic behavior of suspended bridges can be highly nonlinear. Hence, a hanger system which offers a fairly uniform stress distribution in the cables has many advantages.

The present paper studies the dynamics of a beam hanged on a special suspension system, which is composed of pulleys and cables, and called the *block-and-tackle suspension system*. This

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Nomenclature

$u(\xi,\tau)$	dimensionless beam deflection
ξ	dimensionless coordinate
τ	dimensionless time
μ	beam mass per unit length
EI	bending stiffness
L	total length of the beam
r	frequency parameter
$q(\xi,\tau)$	distributed load
q_s	static distributed load intensity
q_0	dynamic distributed load amplitude
f	forcing frequency
$\mathbf{T}$	transformation from active to passive modes
T_f	period of vibration of the forced structure
T	reference time period $(1/f)$
$T_{n,i}$	dimensionless natural period*
f_i	natural frequency*
λ_i	eigenparameter*
$u_i(\xi)$	normalized modal shape function*
F_i	area of the ith normalized shape function*
$R_{d,i}$	deformation response factor of mode i*
η_i	modal displacement*

* Superscript "a" or "p" would correspond to active or passive suspension system, respectively.

effective suspension system was invented by Kolozsváry [10] for supporting tensile roofs [11,12]. It may also be used as a suspension system of footbridges, as suggested in [13], where a deck was suspended to a block-and-tackle suspension system and static analysis of the structure was accomplished. However, the dynamic behavior, which is very important in case of light and slender footbridges, has not been studied yet for such structures.

First the mechanical model is introduced in Section 2. Then in Section 3 the natural circular frequencies and the corresponding modal shape functions are derived for two states of the bridge. One state corresponds to slacked cables, and the other state corresponds to stressed cables. The obtained modal shape functions and frequencies are verified, and the modal decomposition based continuum approach for dynamic simulations is developed in Section 4. The vertical vibration of the structure due to vortex-shedding and passenger traffic is simulated and validated in Section 5. Finally, conclusions are drawn in Section 6.

2. The mechanical model

The mechanical model of the structure is shown in Fig. 1. There is a simply supported *Bernoulli–Euler* beam of length L. Two pulleys are attached to this beam at equal distances. These pulleys divide the beam into three spans of length $\ell = L/3$. The mass of the pulleys, and the friction between the pulleys and their shafts are neglected. There is a rigid upper structure at height h, to which three pulleys are attached, as shown in Fig. 1. A massless, inextensible cable runs through the pulleys. The ends of the cable are fixed to the ends of the beam. The origin of a left-handed coordinate system is at the fixed support of the beam, the x-axis points to the right and coincides with the unloaded, straight axis of the beam,

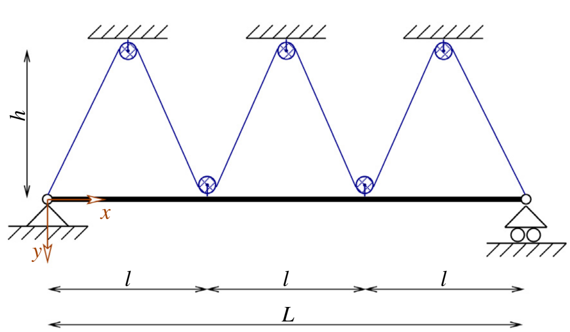

Fig. 1. Model of a beam on a block-and-tackle suspension system.

while the y-axis points downward. The loads act in the $x-y$ plane, and cause uniaxial bending about the z-axis. Lateral and lateral-torsional vibrations, and structural damping are neglected. Small displacements are assumed, $\hat{u}(x,t)$ denotes the vertical deflection of the beam.

3. Natural circular frequencies and modal shape functions

Let us introduce the dimensionless coordinate ξ and time τ as:

$$\begin{aligned} x &= \xi L \quad \rightarrow \quad \tfrac{\partial \xi}{\partial x} = \tfrac{1}{L}, \\ t &= \tau T \quad \rightarrow \quad \tfrac{\partial \tau}{\partial t} = \tfrac{1}{T}. \end{aligned} \tag{1}$$

Here the reference length $L = 3\ell$ is set to the total length of the beam and T is a reference time period which will be fixed later, depending on the studied problem. The beam deflection can be given as function of the dimensionless variables:

$$u(\xi,\tau) = \hat{u}(L\xi, T\tau)/L. \tag{2}$$

Then, partial differential equation that governs the free vibration of a *Bernoulli–Euler* beam [14] yields:

$$\frac{EI}{L^3}\frac{\partial^4 u(\xi,\tau)}{\partial \xi^4} + \frac{\mu L}{T^2}\frac{\partial^2 u(\xi,\tau)}{\partial \tau^2} = 0. \tag{3}$$

Here EI is the bending stiffness of the beam and μ is its mass per unit length. The solution for (3) is searched for in the separated form

$$u(\xi,\tau) = \sum_{i=1}^{\infty} u_i(\xi)\cdot(a_i\cos(2\pi f_i T\tau) + b_i\sin(2\pi f_i T\tau)). \tag{4}$$

Here f_i is the ith natural frequency of the beam, while $u_i(\xi)$ is the corresponding dimensionless *modal shape* function. The coefficients a_i and b_i depend on initial conditions for prescribed shape functions. Substituting (4) in (3) leads to ordinary differential equations (ODEs):

$$\frac{EI}{L^3}u_i^{IV}(\xi) - (2\pi f_i)^2 \mu L u_i(\xi) = 0, \quad i = 1,2,\ldots \tag{5}$$

Here prime denotes differentiation with respect to ξ, hence $u_i^{IV}(\xi) = \mathrm{d}^4 u_i(\xi)/\mathrm{d}\xi^4$.

Let us introduce the *frequency parameter* r of the beam as:

$$r = \frac{T}{L^2}\sqrt{\frac{EI}{\mu}}. \tag{6}$$

Eq. (5) is solved by