Engineering Structures 101 (2015) 494–508

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Nonlinear analytical model of a two-layer wooden beam in a heritage structure

P.L. Cao, Q.S. Yang *, S.S. Law

Beijing Key Laboratory of Structural Wind Engineering and Urban Wind Environment, School of Civil Engineering, Beijing Jiaotong University, Beijing 100044, China

ARTICLE INFO

Article history:
Received 17 December 2014
Revised 20 July 2015
Accepted 27 July 2015
Available online 12 August 2015

Keywords:
Two-layer beam
Wooden
Heritage
Elasticity
Friction
Slip
Tenon
Mortise
Hysteretic loop
Tibetan structure

ABSTRACT

Existing heritage wooden structures in China have a common structural component which composes of layers of wooden beam to resist the vertical applied load. The performance of this type of component under load is not clear particularly under the seismic effect. This paper presents a nonlinear analytical model of a two-layer wooden beam system based on Euler–Bernoulli beam theory. The proposed model takes into account the effect of the friction–slip–shear between the two individual layers with a specified range of slip. There are four deformation scenarios to provide the load resistance based on the state of friction between the two layers and the state of shear at the connector (tenon). The behavior of the tenon with the friction–slip–shear mechanism is significant to the mechanical behavior of the two-layer beam system. This analytical study aims at evaluating the frictional stiffness of a two-layer wooden beam, and an implicit formulation on the frictional stiffness of the system is presented. Its effect on the lateral deformation when subjected to four kinds of loadings is studied. The hysteresis curve is then plotted for different kinds of pseudo-dynamic external loads to illustrate the energy dissipation capacity of this type of joint in Tibetan structures.

1. Introduction

Composite layered systems have been popular in both ancient and modern building structures. They are layered wooden systems or steel–concrete systems which may be glued, nailed or bolted. They have been used widely in floor, beam and beam-column joint systems. This paper studies a cantilevered two-layer wooden beam linked at the free end with a shear connector called tenon.

The beams are usually placed one on top of the other in layers in Chinese wooden heritage structures, and they are connected by a few tenons. Examples are found in Yingxian pagoda and in the wooden frame of ancient Tibetan structures as shown in Figs. 1 and 2. Relative displacement occurs at the interface between two layers of beam, and this interface/contact surface and the shear connector play significant roles in the load resisting mechanism of the layered beam system. The friction–slip at the interface and/or shearing of the tenon gives rise to nonlinear boundary condition for the beam deformation which significantly affects the mechanical behavior of the composite systems [1].

Many analytical researches have been conducted since middle of the last century to study the mechanical behavior of a two-layer beam [2–7]. They were based on the composite beam theory with the Euler–Bernoulli beam and incomplete interaction.

The efficiency of a layered beam generally relies on the connection system at the interface [8,9], and many models have been developed for studying composite layered beam. The two-layer simply supported beam has been modeled as rigidly connected with evenly distributed shear connectors [3–5,10–14]. Others assumed continuously distributed deformable shear connectors along the longitudinal direction of the beam, and there is no friction at the interface between the two beam layers [10,13]. The interaction between the two layers comes from the load–slip characteristics of the interface [3,4,6,10–12]. These studies also share the common feature that the shear stiffness between the two layers of beam is a known constant, and there is no friction between them whatever the position, geometry and material of the connector [5,6,13,14]. Chakrabarti et al. [15] introduced the stiffness of the shear interaction at the interface as a known parameter in a finite element model of composite beam. Kroflic et al. [16] believed the interacting shear and contact-traction between layers is derived from the nonlinear shear contact traction–slip characteristics of the interface. Other scholars obtained the load–slip behavior of the interface from push-out shear test [17–20]. The interacting shear–slip relationship of the finite element proposed by Zona et al. [21] has also been obtained experimentally [22].

---

* Corresponding author.
E-mail address: qshyang@bjtu.edu.cn (Q.S. Yang).

http://dx.doi.org/10.1016/j.engstruct.2015.07.047
0141-0296/

Fig. 1. The layered beam structure in Yingxian pagoda.

The beam layers in heritage wooden structures are, however, usually connected loosely with the tenon at the mortise (a cavity in the beam layers which is slightly larger than the tenon). The gap between the tenon and the beam at the mortise will change under loading, inducing opposite internal forces in the beam layers. The dimension of the gap needs to be taken into account in providing load resistance from the layered beam system. In the study of vertical deformation, Robinson and Naraine [1] discovered that the separation of the interface between the two layers is often small and can be neglected.

This paper presents a new partial slip model for analyzing a two-layer cantilevered beam that takes into account the friction–slip of the interface, the shearing of the tenon and the gap between the tenon and the beams. The model is developed from the equilibrium equations of each layer with an assumed distribution of the frictional stress at the interface between the two layers. Since this composite beam is an important component in modern and heritage wooden structures, its capability for energy dissipation when under seismic excitation would be of interest to general readers. The energy dissipation behavior of the system is then studied with different types of pseudo-dynamic loads.

2. Proposed two-layer beam system

The planar two-layer cantilevered wooden beam model is shown in Fig. 3 and the shear connector between the two layers is shown in Fig. 4. It supports three kinds of external loadings, i.e. M_e, V_e and N_e, which are the bending moment, transverse load and horizontal axial load at the free end. p denotes a uniform transverse load on top of the beam, which may be due to the self-weight of beam and floor slab or from the live load on the slab. The length of the layered beam is L. The deformation of the beam is limited by a wooden stick fit into a hole close to the free end of the two wooden beam layers. The wooden stick and the hole in the beams are called tenon and mortise respectively. Due to fabrication error and the shrinkage distortion of wood, there will be a gap between the mortise in the upper and lower beams and the tenon, denoted as s as shown in Figs. 5–7. Friction and relative longitudinal displacement at the contact surface between the two beams will occur under external loads. When the relative longitudinal displacement is larger than s, the tenon will be subjected to longitudinal shear which contributes to the load resistance of the layered beam. The location, range of slip and the distribution of the friction vary with the changing external loads. Therefore, the model is a good example of a structure with nonlinear boundary conditions.

Fig. 2. The layered beam structure in Tibetan ancient architecture.

In the derivation of the load-deformation relationship for the layered beam, Euler–Bernoulli beam is adopted for the beam system with the following assumptions:

(a) The constituent materials behave elastically in both tension and compression.
(b) Plane sections remain plane and normal to the neutral axis of bending, and the curvatures of the two beams are assumed equal.
(c) Both beams have a unit thickness.
(d) Relative friction and slip can occur at the interface between the beams under load. The relative longitudinal displacement between the two beams has a maximum value of s confined by the tenon at the free end.
(e) The tenon has a rectangular cross-section.
(f) The tenon can rotate within the cavity of the mortise.
(g) The bending rigidity of the layered beams is assumed constant along their lengths.
(h) The tenon is assumed rigid in providing shear resistance to movement of the beams when a specified slip range s at the free end is reached.
(i) The reaction to the compression of the tenon will act at the free end of the beam.
(j) The vertical load acting on the contact surface between two layers of beam follows a trapezoidal distribution.

The load resisting mechanism of the two-layer beam system consists of a fully sticky stage and a partial slip stage defined by the relative longitudinal displacement and the frictional force along the interface. Each of these stages can be further divided into two scenarios with the tenon subjected and not subjected to shear.

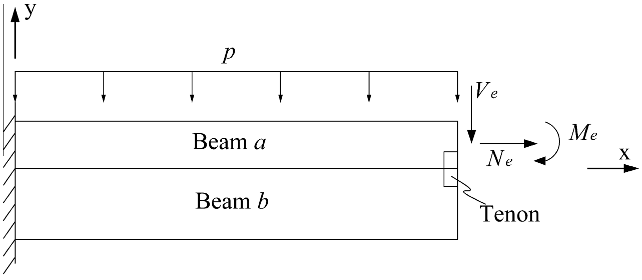

Fig. 3. A cantilevered two-layer beam.