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Automatic cross-section classification and collapse load evaluation for steel/aluminum thin-walled sections of arbitrary shape

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ABSTRACT

We present a computational procedure for evaluating the collapse load and assessing the cross-section classification of thin-walled sections of arbitrary shape on the basis of Eurocode prescriptions. The procedure is based on two algorithms which address separately the rigid-plastic model adopted by the Eurocode for ordinary steel cross-sections and arbitrary uniaxial constitutive laws typically used for stainless steel and aluminum. Both algorithms are based on a polygonal description of the cross section boundary so that integrals extended to the section domain are conveniently expressed as algebraic sums depending upon the coordinates of the section vertices. Accordingly, a further algorithm is illustrated in order to automatically convert the plate and node model adopted by Eurocode to a polygonal description of the section geometry. The numerical effectiveness of both algorithms is assessed with reference to an I-shaped, a Z-shaped and a RHS cross sections.

1. Introduction

Both the design and the verification of a structure have to take into account not only the attainment of yield or fracture conditions, but also the occurrence of buckling phenomena, as they can cause collapse of the structure for stress levels lower than the strength of material supposed to act on the whole section. In particular, buckling analysis is undoubtedly fundamental for thin-walled beams, as the buckling load is inversely proportional to the slenderness of the beam. Despite the high values of yield and ultimate stress guaranteed by steel or aluminum and their iso-resistant behavior, compression on the plates composing the cross-section of a thin-walled steel/aluminum beam is likely to determine the attainment of local buckling conditions for relatively low values of the applied load.

Due to the extraordinarily large employment of structural thin-walled steel elements subject to axial load and bi-axial bending, thus likely to undergo local buckling, it is easy to conclude that a thorough study on this issue is particularly relevant in order to ensure the fulfillment of safety and reliability requirements.

Among several strategies proposed in the past for nonlinear analysis of thin walled beams [1–3] we mention the Generalized Beam Theory (GBT) since it has been recently object of a renewed interest [4–6]. Complementary researches regard the analysis of the sectional behavior although they are usually focused on applications to specific cross-section shapes. For example, the analysis of H-sections behavior underlines the relevance of interactive effects, especially for complex load patterns [7,8]. Interaction effects on constituent plates have also been considered by Zhou et al. [9] in order to determine enhanced class 3 slenderness limits for square and hollow sections in compression.

Gardner and Theofanous [10] have shown the advantages associated with the application of a new approach, called Continuous Strength Method, based upon the adoption of an experimentally determined curve, relating the strain at which local buckling occurs to the slenderness of the cross section.

In spite of their theoretical reliability, these approaches may prove to be hardly applicable to practical design necessities, especially when a large number of different elements has to be taken into account. Furthermore, as previously discussed, some of the existing methodologies, though accurate and sophisticated, refer to cross-sections of specific shapes, whereas it would be clearly preferable to set up a unique strategy able to encompass an arbitrary cross-section geometry.

A practical answer to the aforementioned necessities is provided by design regulations. Most of them adopt a cross-section classification which is fundamentally based upon the capacity of the cross section to fully develop a plastic hinge before local buckling occurs. In practice, the section classes are evaluated by comparing the length-to-thickness ratios of the single plate composing the cross-section with suitable functions, which

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depend on the material properties, on the constraint conditions of the plates and on the normal stress distribution acting on the cross-section [11].

Such approaches are followed, with limited differences, both by American regulations, e.g. ANSI/AISC 360-10 [12], and by Eurocodes [13,14]. Though substantially based upon the same approach, Eurocodes seem to provide a more detailed and versatile description of the phenomenon under examination. In particular, unlike ANSI/AISC 360-10, the classification procedure suggested by Eurocodes does not refer to specific cross-section shapes [15,16]. Actually, the section is conceived as an arbitrary collection of rectangular plates so that, on the basis of their mutual constraint conditions, the procedure adopted by Eurocodes can be applied to any cross section and will be addressed in below.

Actually, the intent of our work is not to investigate on the reliability of the procedure provided by building regulations, but rather to implement an automatic procedure for thin-walled cross-section classification useful for design purposes. A similar study has been conducted by Rugarli [17]; though limited to the classification of I- or H-shaped cross-sections, it is computationally very efficient in case a high number of strength checks has to be conducted for each section. On the contrary, the procedure described in the present paper has been formulated with the specific intent of being applicable to cross-sections of arbitrary shape and capable of detecting the collapse load for any cross-section class.

Specifically, the nonlinear analysis preliminary to class 1 or 2 grading is carried out in the present paper on the basis of two separate algorithms depending on material constitution. Actually, a rigid-perfectly plastic behavior is prescribed by Eurocodes for ordinary steel so that the Nelder–Mead simplex method [18] has been adopted. Conversely, for stainless steel and aluminum, nonlinear constitutive laws are suggested in the literature; for this reason the secant method [19,20] enhanced with the fiber-free approach [21,22] has been adopted. The same method has also been used for the elastic analysis required for class 3 and 4 sections.

The domain integrals required by the secant approach are computed analytically, thanks to the fiber-free approach, by considering a boundary representation of the section. On the contrary Eurocode addresses sections by a plate and node model, i.e. as a discrete collection of nodes connected by plates which represent webs and flanges of the section. For this reason, in order to obtain a fully automated cross-section classification procedure, an algorithm which allows one to obtain the polygonal description of the cross-section starting from the *plate and node* model has been developed.

The paper is organized as follows: in Section 2 we formulate the equilibrium problem to be solved and motivate the adoption of two different procedures to solve the sectional equilibrium equation. In Section 3 a different formulation of the sectional equilibrium is described so as to properly employ the simplex method. In Section 4 we present the automatic procedure that is used to switch from the *plate and node* model of the section to a polygonal representation of its boundary. Finally, three numerical examples are reported in Section 5 for classifying an I-shaped, a Z-shaped and a rectangular hollow section. While for the I-shaped section a comparison can be performed with available results in the literature [17], the other examples have been considered intentionally to show the applicability of the proposed approach to more general cases.

2. The sectional analysis procedure

The Eurocodes rules for cross-section classification require the evaluation of the normal stresses σ attained at the end points of each plate of the section subject to its ultimate load. Since the cross-section is subject to axial force N and bending moments M_x and M_y, sectional ultimate load is not unique but depends on the combination of internal forces acting on the section.

In order to define the ultimate load of the section, a load path is defined as follows: the internal forces which act on the section are collected in the vector $\mathbf{f} = (N, -M_y, M_x)^t$. It is assumed that $\mathbf{f}$ can be additively decomposed as sum of two components, $\mathbf{f}_d$ and $\mathbf{f}_l$, which respectively denote the internal forces associated with dead and live loads. In this way one is free to decide which part of the internal forces need to be amplified according to a load parameter λ. For instance, a load path characterized by the amplification of the internal forces $\mathbf{f}_l$ is defined as:

$$\mathbf{f}(\lambda) = \mathbf{f}_d + \lambda \mathbf{f}_l \tag{1}$$

The components of the internal force vector $\mathbf{f}$ are evaluated as a function of the normal stresses $\sigma(\mathbf{r})$ acting at the points $\mathbf{r} = (x, y)^t$ of the cross-section Ω by means of the integrals:

$$N = \int_\Omega \sigma(\mathbf{r})\, dA \quad \mathbf{M}_r^\perp = (-M_y, M_x)^t = \int_\Omega \sigma(\mathbf{r})\, \mathbf{r}\, dA \tag{2}$$

Introducing the vector $\boldsymbol{\rho} = (1, x, y)^t$ to simplify the notation, equilibrium of the section is formulated as:

$$\mathbf{f}(\lambda) = \int_\Omega \sigma(\mathbf{r})\, \boldsymbol{\rho}\, dA = \mathbf{f}_d + \lambda \mathbf{f}_l \tag{3}$$

Due to the nonlinear constitution of the material which composes the section, Eq. (3) is nonlinear and its solution for a given value of the vector $\mathbf{f}$ requires an iterative procedure. Some algorithms for solving Eq. (3) in the case of ultimate strength analysis of ordinary and prestressed reinforced concrete sections can be found in [19,20,23,24]. In this case one has to determine λ in (3) so that assigned ultimate values of strain are attained in the section.

Differently from the constitutive assumptions commonly adopted for the limit state analysis of sections, the procedure described in EC3 for cross section classification is based on a rigid-plastic material, since this is commonly used in the context of limit analysis. This assumption implies that the material is considered to be indefinitely ductile so that no ultimate strain is assigned.

Clearly, the use of a tangent approach [23] to the solution of (3) is precluded since the indefinite flat branch of such constitutive law produces a singular cross-section stiffness matrix. Also, the rigid portion of such stress–strain law makes inapplicable the secant algorithms described in [19,20,24], since an infinite value of secant stiffness is associated with the points along the neutral axis. Consequently, a different iterative algorithm needs to be applied in presence of rigid-plastic constitutive assumptions.

As a matter of fact the rigid-plastic constitutive law is suggested by EC3 only for steel cross sections of class 1 and 2 since steel sections are assumed to belong to classes 3 and 4 on the basis of the elastic stress limit. However, for stainless steel and aluminum, the yield strength and the elastic limit are not always clearly defined so that constitutive laws more refined than the classical elastic-perfectly plastic or the rigid-plastic ones are usually adopted for these materials. In this case the yield strength and the elastic limit are conventionally assigned by EC3 though the key role played by constitutive modeling of stainless steel and aluminum towards their classification has been recently established [10].

On account of the previous considerations two alternative algorithms are employed in the proposed automatic cross-section classification: (i) the Nelder–Mead simplex method [18] is addressed for the rigid-plastic constitution assumed by EC3 for steel cross sections; (ii) the secant approach formulated in [19,20] and enhanced with the integration formulas presented in [21,22] is considered for an arbitrary nonlinear constitutive law. Being this