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# Analytical solutions for flexural design of hybrid steel fiber reinforced concrete beams

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#### ABSTRACT

Hybrid reinforced concrete (HRC) is referred to as a structural member that combines continuous reinforcement with randomly distributed chopped fibers in the matrix. An analytical model for predicting flexural behavior of HRC which is applicable to conventional and fiber reinforced concrete (FRC) is presented. Equations to determine the moment–curvature relationship, ultimate moment capacity, and minimum flexural reinforcement ratio are explicitly derived. Parametric studies of the effect of residual tensile strength and reinforcement ratio are conducted and results confirm that the use of discrete fibers increases residual tensile strength and enhances moment capacity marginally. However improvements in post-crack stiffness and deformation under load is substantial in comparison to conventional steel reinforcement. Quantitative measures of the effect of fiber reinforcement on the stiffness retention and reduction of curvature at a given applied moment are obtained. The approach can also be presented in a form of a design chart, representing normalized moment capacity as a function of residual tensile strength and reinforcement ratio. Numerical simulations are conducted on the steel fiber reinforced concrete (SFRC) and HRC beam tests from published literature and the analytical solutions predict the experimental flexural responses quite favorably.

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### 1. Introduction

For more than forty years FRC has been used in many construction applications such as slabs on grade, industrial floors, tunnel linings, precast and prestressed concrete products. Use of discrete fibers significantly improves fracture toughness, ductility, fatigue resistance, as well as tensile and shear strength. Recent advances in performance of FRC have been based on a sufficiently high fiber content ( $0.5\% < V_f < 1\%$ ) to gain significant ductility and strength. A fiber content of 0.75% without stirrups is considered sufficient to achieve the equivalent ultimate resistance of a conventional RC flexural member with stirrups [1]. The use of fiber also enhances the behavior at service life conditions by increasing the stiffness and residual strength in the serviceability loading stage by means of restraining the crack opening and limiting excessive deformations [2]. This has led to development of structures such as elevated SFRC slabs and precast tunnel lining segments that use a hybrid reinforcement approach [3–5]. Portions of the conventional reinforcement are replaced by steel fibers in most parts to address the flexural capacity. In the case of elevated slabs only a small amount of reinforcement is needed along the column strips to prevent progressive failure, while the amount of rebar in precast segmental sections is substantially reduced.

The enhancement in the load capacity and ductility depend on the fiber parameters such as type, shape, aspect ratio, bond strength and volume fraction [6]. Tensile characteristics are defined in terms of strain softening and hardening, and within the strain softening category, sub-classes of deflection-softening and -hardening may be defined based on the behavior in bending [7]. Several building codes provide guidelines on design with FRC materials [8-11]. Combinations of FRC and rebars or welded wire mesh may be used to meet the strength criteria, hence HRC is referred to as a section that combines a continuous reinforcement with randomly distributed chopped fibers. Many available models for FRC [12–15] require a strain compatibility analysis of the layered beam section in order to obtain moment capacity, which may be impractical for general users. Development of a unified approach for both continuous and discrete reinforcements is therefore needed.

Post-cracking tensile behavior of FRC materials have been simulated by either a stress-strain ( $\sigma$ - $\varepsilon$ ) relationship in a smeared crack continuum model, or a stress-crack width ( $\sigma$ - $\omega$ ) discrete model using non-linear fracture mechanics. The original discrete







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## Notation

$A_s$	area of steel rebar
b	beam width
$B_{1-5}$	coefficients for neutral axis depth ratio in Table 5
$C_{1-11}$	coefficients for normalized moment in Table 5
d	effective depth at location of steel rebar
Ε	elastic tensile modulus of concrete
Ec	elastic compressive modulus of concrete
E <sub>s</sub>	elastic modulus of steel
fc	cylindrical ultimate compressive strength of concrete
f	stress components in stress diagram
F	force components in stress diagram
G <sub>1</sub> , G <sub>2</sub>	coefficients for minimum flexural reinforcement in Eq.
	(21)
h	full height of a beam section or height of each compres-
	sion and tension zone in stress diagram
Κ	effective flexural stiffness of a beam section
k	neutral axis depth ratio
Μ	moment
$M_n$	nominal moment capacity
$M_u$	ultimate moment
п	modulus ratio $(E_s/E)$
R	coefficient of resistance
у	moment arm from force component to neutral axis
α	normalized depth of steel reinforcement $(d/h)$
β	normalized tensile strain $(\varepsilon_t/\varepsilon_{cr})$
$\beta_1$	coefficient for the depth of ACI rectangular stress block
3	strain
ε <sub>c</sub>	concrete compressive strain
$\varepsilon_{c0}$	concrete compressive strain at peak stress
$\varepsilon_{ctop}$	concrete compressive strain at top fiber
$\varepsilon_t$	concrete tensile strain
$\varepsilon_{tbot}$	concrete tensile strain at bottom fiber
$\phi$	curvature
γ	normalized concrete compressive modulus $(E_c/E)$
κ	normalized steel yield strain $(\varepsilon_{sy}/\varepsilon_{cr})$
λ	normalized compressive strain $(\varepsilon_c/\varepsilon_{cr})$
$\lambda_{R1}$	normalized compressive strain at the end of elastic re-
	gion 1
$\mu$	normalized residual tensile strength $(\sigma_p/\sigma_{cr})$
$\mu_{crit}$	the critical normalized residual tensile strength that
	change deflection-softening to deflection-hardening
$\rho$	steel reinforcement ratio per effective area
$ ho_{bal}$	steel reinforcement ratio per effective area at balance
0	Idilule
$ ho_g$	steer remiorcement ratio per gross area

	$ ho_{g,bal}$	failure
	Øg min	minimum flexural reinforcement per gross section
	$\rho_{\sigma \min rc}$	minimum flexural reinforcement per gross section for
	7 5,1111,10	conventional reinforced concrete
	$\rho_{min}$	minimum flexural reinforcement ratio per effective
	,	section
	$\rho_{min.rc}$	minimum flexural reinforcement ratio per effective
	•	section for conventional reinforced concrete
	$\sigma$	concrete stress
	$\sigma_c$	concrete compressive stress
	$\sigma_p$	residual tensile strength
	$\sigma_t$	concrete tensile stress
-	ω	normalized concrete compressive yield strain $(\varepsilon_{cy}/\varepsilon_{cr})$
	χ	normalized steel strain ( $\varepsilon_s/\varepsilon_{cr}$ )
	Subscript	S
	1	at stage 1, elastic compression-elastic tension
	21	at stage 2.1, elastic compression-residual tension, steel
		is elastic
	22	at stage 2.2, elastic compression-residual tension, steel
		is yield
	31	at stage 3.1, plastic compression-residual tension, steel
		is elastic
_	32	at stage 3.2, plastic compression-residual tension, steel
¢.		is yield
	c1	elastic compression zone 1 in stress diagram
	с2	plastic compression zone 2 in stress diagram
	cr	at first cracking
	си	at ultimate concrete compressive strain
	cy	at concrete compressive yielding
	1	at stage <i>i</i> of normalized concrete compressive strain and
		tensile steel condition
	S	refer to steel
	sy	at steel yielding
-	11 +7	elastic tension zono 2 in stress diagram
	ιZ tu	at concrete ultimate tensile stain
	cu cu	at concrete ultimate compressive strain
t	cu m	at concrete compressive strain approach infinity
-	$\sim$	at concrete compressive strain approach minity

Superscripts

normalizing symbol

crack approach by Hillerborg et al. [16] has been modified by many researchers [17–19]. It does not address crack formation and propagation, but instead uses a stress-crack width  $(\sigma - \omega)$  response as an input parameter in the post peak tensile zone [20,21]. A representative volume element of a cracked section of a flexural beam with length  $L_p$  and depth *h* is shown in Fig. 1. The section is characterized by compression and tensile zones. The tensile zone is represented by two regions; an elastic tensile strain as well as a bridged crack in opening mode. The stresses carried by fibers across the crack in tension are represented as a function of crack opening and the method is widely used in simulation and design of quasi-brittle materials [11,22,23]. One of the main parameters of these models is a characteristic length parameter defined as  $L_p$ , which prevents mesh dependency of the results in finite element models as it relates the crack width to strain [24,25]. In smeared crack models, characteristic length parameter determines the width of localization and prevents snap-back and other numerical instabilities [26]. In the present paper the length of localization zone has been used as a constant length parameter that affects the postpeak descending response of the load deformation curve where cracks are localized. The  $\sigma$ - $\varepsilon$  approach is more suitable for HRC elements since distributed cracking and tension stiffening are expected [27]. For example application of superposition to add the contribution of reinforcement and fibers by updating the stress crack width relationship in the tensile zone of multiple cracks in under-reinforced flexural sections is challenging. Furthermore, reinforcement ratio affects rebar stress and affects crack opening which will in turn affect fiber phase's contribution.

Development of a serviceability design approach based on deflection, ductility or allowable stress would require the computation of load capacity of a cracked section based on a given curvature or crack width. Such solutions would keep track of the strain Download English Version:

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