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Probabilistic approach for modelling the load-bearing capacity of glued laminated timber

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ABSTRACT

In this paper, a probabilistic approach for modelling the load-bearing capacity of glued laminated timber is presented. The specific characteristic of this approach is that, at first, timber boards are simulated according the natural growth characteristic of timber. Subsequent, glued laminated timber beams are virtually composed out of the simulated timber boards. Thereby, every kind of fabrication procedure, such as the length of the timber boards or the beam dimensions, can be recreated. Afterwards, a numerical model is used to estimate the load-bearing capacity, the bending stiffness and the type of failure of the simulated GLT beams. To ensure the quality of the numerical model it is validated with 24 GLT beams with a precisely-known beam setup; a wide agreement between the measured and the estimated material properties is identified. For a probabilistic investigation of different input parameters a Monte Carlo simulation is performed. The application of the presented approach is illustrated on selected examples (size effect, the quality of finger joint connections and grading criteria).

1. Introduction

Timber is a natural grown material. Thus, compared to other building materials, timber properties demonstrate higher variability. The variability is pronounced between different structural elements (e.g. through different growth conditions) and within single elements (through knots and knot clusters). In Nordic spruce timber specimens knot clusters are distributed over the length of a timber board with rather regular longitudinal distances. Within glued laminated timber (GLT) the variability is slightly reduced through homogenisation (laminating effect). This means that local weak sections, such as knot clusters are distributed more homogenised than in solid timber. Thus, the influence of single defects is reduced and local weak sections are reinforced by adjacent lamellas (for a detailed description see e.g. [14,46]). However, because of the relatively regular distance between knot clusters the joint appearance of knot clusters from different lamellas in the same cross section is quite frequent.

The load bearing capacity of GLT, or rather the characteristic value of the bending strength $f_{m,g,k}$ is investigated since more than 30 years within numerous different studies [4]. The outcome of the majority of these studies is an empirical equation to predict $f_{m,g,k}$ based on the properties of the source material; e.g. the characteristic value of the tensile strength of the lamellas $f_{t,0,l,k}$ or the characteristic value of the bending strength of finger joint connections $f_{m,j,k}$. From a scientific perspective those equations are often unreproducible. One example is the equation given in the current version of the EN 14080 [13], that contains altogether 7 empirical values:

$$f_{m,g,k} = -2.2 + 2.5 f_{t,0,l,k}^{0.75} + 1.5 (f_{m,j,k}/1,4 - f_{t,0,l,k} + 6)^{0.65} \quad (1)$$

The basis for the empirical equations are in general test results or model simulations. Regarding the huge number of influencing parameters the use of simulation methods have been established as the more efficient within the last decades. Examples are the *Model of Foschi and Barrett* [23], the *Prolam model* [2,29] or more recently the *Karlsruher Rechenmodell* [12,5,3]. Further, investigations on probabilistic models were performed by e.g. [27], [40][1], [37]. Detailed information about the individual models are given in the mentioned publications.

The quality of the simulation models has improved since the first approach developed by Foschi and Barrett [23]. Thereby in particular the development of the *Karlsruher Rechenmodell* has to be mentioned. However, there are still some opportunities for improvement such as (a) the use of more efficient strength and

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[1] Cited in [47].

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stiffness related indicators, (b) an improvement of the probabilistic description of timber boards, or (c) an improvement of the material models. Detailed explanations about the improvements are given in the corresponding sections. Furthermore, it has to be mentioned that the quality of the numerical models, in respect to predicting the load-bearing capacity of individual GLT beams with well-known local material properties (i.e. GLT beams where the exact position of each particular knot cluster and each particular finger joint connection were known), has not been validated in detail. An exception is the *Karlsruher Rechenmodell*, where an updated version of the model was validated in Ehlbeck and Colling [10,11], who tested altogether nine GLT beams, where the above-mentioned information of the lowest two lamellas is known. However, only in two GLT beams, a finger joint connection (FJ) was placed in the highest loaded area – both failed within the FJ. As a result, the quality of the numerical models, in terms of considering varying material properties and detecting the type of failure, is not completely proved yet.

In the present paper, a model for the probabilistic representation of the material properties of GLT is developed that considers the natural growth characteristic of timber. Further, 24 GLT beams with well-known local material properties are fabricated and tested in order to validate the model. In the last part, the application of the probabilistic model is illustrated on selected examples. The examples include, among others, the investigation of the influence of the size effect and the quality of finger joint connections on the load-bearing capacity.

2. GLT model

In this research project a model for the probabilistic representation of the material properties of GLT (referred to as *GLT model*) is presented that considers the natural growth characteristic of timber. The principle idea of the developed model is comparable to existing models; i.e. GLT beams having a specific beam set-up are simulated and their load-bearing capacity will be estimated.

The structure of the GLT model is illustrated in Figs. 1 and 2; it contains four independent sub-models: (1) simulation of timber boards, (2) fabrication of GLT beams, (3) allocation of material properties, and (4) a numerical model for the estimation of the load-bearing capacity. In the following sections, these sub-models are introduced.

2.1. Simulation of timber boards

Timber boards are simulated using the probabilistic model introduced in [21]. The specific characteristic of this model is that the natural growth characteristics of timber is considered; i.e. the position and the characteristics of knot clusters are simulated. The model includes a representation of the geometrical setup, as well as a hierarchical representation of two strength and stiffness related indicators. One indicator to describe the mean material properties of the timber board and one indicator to describe the local strength and stiffness reduction due to knots. The model was developed, to model timber boards of two strength grades L25 and L40 (Norway spruce), graded by the GoldenEye-706 grading device [26]. Lamellas of this strength grade require a minimum characteristic tensile capacity of 14.5 MPa and 26.0 MPa, respectively (EN 14081-4 [9]).

The geometrical setup of timber boards is described with the distance d between weak sections (denoted WS). A WS is defined as a knot cluster with a total knot area ratio tKAR $\geqslant 0.1$. The tKAR-value describes the ratio between the projected knot area within a length of 150 mm and the cross-section area, overlapping knots are counted only once (e.g. [30]), having a constant length $l_{WS} = 150$ mm. d is defined as the distance between the mid-points of two adjacent WS.

Due to growth characteristics of Norway spruce combined with the cutting process the distance between knot clusters that appear in the timber boards might be best represented by the gamma distribution. However, in [21] the length of a WS was assumed to be constant ($l_{WS} = 150$ mm), thus the minimal distance between two adjacent WS is $d_{min} \geqslant 150$ mm. Accordingly a shifted gamma distribution is used to describe d; ν and k are the model parameter of the gamma distribution:

$$f(d) = \frac{\nu(\nu(d-l_{WS}))^{k-1}}{\Gamma(k)} e^{-\nu(d-l_{WS})} + l_{WS} \quad \text{for} \quad l_{WS} \leqslant d \leqslant \infty \tag{2}$$

Between the two strength grades only marginal differences are identified, that correspond to the findings presented in [6]. Consequently d is identified for all specimens, independent of the strength grade.

The two strength and stiffness related indicators are described using a log-normal distribution. One indicator, the dynamic modulus of elasticity ($E_{dyn,F}$), is essential to model the material properties of the defect-free timber. Whereas the other one, the tKAR-value, is used to model the local strength and stiffness reduction through knots.

The tKAR-value of the WS is described according to Eq. (3) by a hierarchical model having two hierarchical levels [33,34]: Meso-scale and micro-scale. The tKAR-value of the WS j in a board i (tKAR_{ij}) is represented as a truncated lognormal random variable. μ is the logarithm mean tKAR of all WS within a sample of boards, considered to be deterministic. The meso scale $\tau_i \sim N(0, \sigma_\tau)$ describes the variability of a single board within a sample of boards. The micro scale $\epsilon_{ij} \sim N(0, \sigma_\epsilon)$ describes the variability within one board.

The distribution of the tKAR-value has to be truncated in the upper part, due to the definition of the tKAR-value (the tKAR-value of every board section has to be within the interval [0,1]). Furthermore, an upper limit $\text{tKAR}_{\text{limit}}$ to simulate the grading process can be introduced. However, it has to be considered that the grading process will not be perfect and thus there is always a certain probability that the tKAR-value of WS exceeds the defined threshold.

$$\text{tKAR}_{ij} = \exp(\mu + \tau_i + \epsilon_{ij}) \quad \text{with} \quad \tau_i + \epsilon_{ij} \leqslant \ln(\text{tKAR}_{\text{limit}}) - \mu \tag{3}$$

The second parameter $E_{dyn,F}$ represents the mean value over the entire timber board. Thus only one hierarchical level, the meso-scale $\tau_i \sim N(0, \sigma_\tau)$, is needed:

$$E_{dyn,F} = \exp(\mu + \tau_i) \tag{4}$$

The parameter for the probabilistic model [21] are summarized in Table 1. For both strength grades only marginal correlations between the parameter was identified; here the correlations are not considered.

With the presented model, timber boards of any board length can be simulated. Through the hierarchical modelling of tKAR, the weakest knot cluster within a timber board (tKAR_{max}) depends on the board length. In this part of the simulation process, over-length timber boards are simulated. They are sawn to a specific length in the second sub-model (Fabrication of GLT).

2.2. Fabrication of GLT

In the second step, the simulated timber boards are virtually finger jointed to form endless lamellas, which are then cut to the specific beam length, and glued together to GLT beams. Thereby, in principle every kind of fabrication procedure can be recreated. In the present study, the timber boards are simulated in