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Unified rational formula for pre-cracking torsional stiffness of solid and hollow reinforced concrete members

Chyuan-Hwan Jeng [a,*], Min Chao [b]

[a] Department of Civil Engineering, National Chi Nan University, Nantou, Taiwan
[b] Department of Civil Engineering and Geomatics, Cheng Shiu University, Kaohsiung, Taiwan

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ABSTRACT

It is generally assumed that the elastic torsional stiffness of structural reinforced concrete (RC) members that are subjected to a torsional moment less than the cracking torque can be accurately estimated, despite a lack of adequate experimental or theoretical examination. The softened membrane model for torsion (SMMT), which has been validated experimentally, provides accurate estimates of pre-cracking torsional stiffness for solid and hollow RC members. However, the SMMT requires an iterative solution algorithm, and is thus inconvenient for design purposes. This paper presents a simplification of the SMMT and proposes a unified rational formula for the direct calculation of the initial torsional stiffness of solid and hollow RC members. The proposed formula predicts the initial torsional stiffness of nine hollow RC beam specimens almost perfectly. When used to calculate the stiffness of 147 solid and hollow RC beam specimens, the proposed formula is found to be an almost perfect simplification of the SMMT in terms of initial torsional stiffness. However, the elastic stiffness values of the 147 specimens deviate from the formula-calculated values by −40% to +50%, suggesting the values may not be as accurate as commonly presumed. It is also shown that the proposed formula can combine with an existing $T_{cr} - \theta_{cr}$ formula to provide a precise bi-linear simplification of the nonlinear pre-cracking torque-twist curves for solid and hollow RC members; this can then be conveniently used in the nonlinear finite element analyses needed in performance-based engineering.

1. Introduction

Three-dimensional linear or nonlinear finite element analyses of structures, which are now routine in structural design practice, requires values for the torsional stiffness of the reinforced concrete (RC) members. For this purpose, the elastic values of torsional stiffness are often assumed, and then used in the analyses of structural RC members that are subjected to a torsional moment that is less than the cracking torque; structural members in RC frame structures, for example, are often designed to undergo a limited torsional moment less than cracking torque. However, although the use of an assumed elastic torsional stiffness appears too logical to need examination, there are, in fact, theoretical and experimental difficulties with the precise examination of this assumption. As a consequence, the routine use of elastic torsional stiffness for RC members has rarely, if ever, been examined.

The theoretical difficulty in examining this assumption is that, with the exception of elastic theories [1,2], there are no rational torsion theory that can predict the pre-cracking torque-twist responses of RC members. The rational softened truss model (STM) [e.g., 3–10] can only predict post-cracking responses. The historical review of torsion theories for structural concrete can be found, for example, in references [11,12]. The experimental difficulty is illustrated by the significant variations in the experimental cracking-twist data for RC beam specimens found in the literature [13,14], which are a reflection of the difficulty of measuring the small deformations in the torsion tests of concrete beams.

These theoretical and experimental difficulties have recently been overcome by a series of theoretical and experimental investigations based on a new rational theory called the softened membrane model for torsion (SMMT). This theory successfully incorporates the effect of concrete tensile stress, and thus is capable of predicting both the pre- and post-cracking torque-twist responses of solid RC elements [13,15–18]. On the basis of a new experimental project that tested nine large hollow RC beam specimens [19,20], the original SMMT for solid RC members has been modified and extended to hollow RC members, resulting in a

* Corresponding author. Tel.: +886 492910960x4710; fax: +886 492918679.
E-mail addresses: chjeng@ncnu.edu.tw (C.-H. Jeng), mchao@csu.edu.tw (M. Chao).

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Nomenclature

A	variable as defined in Step 1 in Fig. 7
A_o	area enclosed by the centerline of shear flow
$A_{o,cr}$	A_o at cracking
A_l	total cross-sectional area of longitudinal steel bars
A_t	cross-sectional area of one transverse steel bar
A_c	cross-sectional area bounded by the outer perimeter of the concrete
a	$(A_l/p_0) + (A_t/s)$, variable as shown in Eqs. (2)–(4)
B	variable as defined in Eq. 27 in Fig. 3(a) or as defined in Step 1 in Fig. 7
b	$(A_l/p_0) - (A_t/s)$, variable as shown in Eqs. (2)–(4); width of the rectangular cross-section as illustrated in Fig. 1(a) when associated with the depth-to-width ratio r
C	variable as defined in Step 1 in Fig. 7; variable as defined in Eq. (14)
c	$1/(\varepsilon_2 - \varepsilon_1)$, variable as shown in Eqs. (2)–(4); depth of neutral axis
d	$\bar{\varepsilon}_1 + \bar{\varepsilon}_2$, variable as shown in Eqs. (2)–(4)
E	t_d/E_s, variable as shown in Eqs. (2)–(4)
E_s	elastic modulus of steel bars
f'_c	cylinder compressive strength of concrete
f_{cr}, ε_{cr}	cracking stress and strain of concrete
f_l, f_t	smeared (average) steel stress in longitudinal and transverse directions, respectively
f_s	smeared (average) stress of steel bars
f_y, ε_y	yield stress and strain of bare steel bars
f_{ly}, f_{ty}	yield strength of longitudinal and transverse reinforcing bars, respectively
G	shear modulus of concrete as used in Eq. (14)
H	variable as defined by Eq. (4b)
h	depth of the rectangular cross-section, as illustrated in Fig. 1(a)
jd	internal moment arm in bending of reinforced concrete members
k_{1c}	ratio of the average compressive stress to the peak compressive stress in the concrete struts, taking into account the tensile stress of concrete
k_{1t}	ratio of the average tensile stress to the peak tensile stress in the concrete struts
M	bending moment
p_o	perimeter of the centerline of shear flow
$p_{o,cr}$	p_o at cracking
p_c	perimeter of the outer concrete cross section
q	shear flow
Q	variable as defined in Eq. 15 in Fig. 2(b)
r	h/b; depth-to-width ratio of the rectangular cross-section
s	spacing of transverse hoop bars
t	wall thickness of the hollow cross-section
T	torque
T_{cr}	cracking torque
T_i	terminal torque of the initial $T-\theta$ straight line
t_d	thickness of shear flow zone
$t_{d,cr}$	t_d at cracking
$t_{d,\text{Solid}}$	t_d- value calculated using the SMMT-S Eq. 15 in Fig. 2**(b)**
$t_{d,cr,\text{Solid}}$	$t_{d,cr}$ of a representative solid section as calculated in Step 1 in Fig. 7
$t_{d,cr,\ \text{Hollow}}$	$t_{d,cr}$ for hollow members as used in Step 2 in Fig. 7
α_2	fixed angle, angle of applied principal compressive stress (2-axis) with respect to longitudinal steel bars (l-axis), as illustrated in Fig. 1(b)
β	deviation angle as calculated in Eq. 22 in Fig. 3(a); St. Venant's coefficients as used in Eq. (14)
ε_0	concrete cylinder strain corresponding to peak cylinder strength f'_c
$\varepsilon_1, \varepsilon_2$	smeared (average) biaxial strain in 1- and 2-directions, respectively
$\varepsilon_{1,cr}, \varepsilon_{2,cr}$	$\varepsilon_1, \varepsilon_2$ at cracking, respectively
$\bar{\varepsilon}_1, \bar{\varepsilon}_2$	smeared (average) uniaxial strain in 1- and 2-directions, respectively
$\bar{\varepsilon}_{1,cr}$	$\bar{\varepsilon}_1$ at cracking
$\bar{\varepsilon}_{1s}, \bar{\varepsilon}_{2s}$	uniaxial surface strain in 1- and 2-directions, respectively; $\bar{\varepsilon}_{1s} = 2\bar{\varepsilon}_1$, and $\bar{\varepsilon}_{2s} = 2\bar{\varepsilon}_2$
$\varepsilon_l, \varepsilon_t$	smeared (average) biaxial strain in l- and t-directions of steel bars respectively
$\bar{\varepsilon}_l, \bar{\varepsilon}_t$	smeared (average) uniaxial strain in l- and t-directions of steel bars respectively
$\bar{\varepsilon}_n$	smeared (average) uniaxial yield strain of steel bars
$\bar{\varepsilon}_s$	smeared (average) uniaxial strain of steel bars
ε_{sf}	smeared (average) strain of steel bars that yield first, taking into account Hsu/Zhu ratios
ϕ	curvature of the concrete struts along 2-direction
φ	curvature of the concrete struts along 1-direction
γ_{21}	smear (average) shear strain in 2-1 coordinate
γ_{lt}	smear (average) shear strain in the l-t coordinate of steel bars
η	multiplier factor for the average tensile and compressive stresses of concrete, as defined in Eqs. 20 and 23 in Fig. 3(a)
λ	multiplier factor for the pre-cracking stiffness of the tensile stress–strain relationship of concrete, as defined in Eq. 25 in Fig. 3(a)
μ	multiplier factor for the peak-stress strain of the tensile stress–strain relationship of concrete, as defined in Eq. 25 in Fig. 3(a)
σ_1^c, σ_2^c	smeared (average) normal stresses of concrete in 1- and 2-direction, respectively
$\sigma_{1,cr}^c, \sigma_{2,cr}^c$	σ_1^c, σ_2^c at cracking, respectively
σ/ε	abbreviation symbol for the parameter $(\sigma_1^c - \sigma_2^c)/(\varepsilon_1 - \varepsilon_2)$
τ_{21}^c	smeared (average) shear stress of concrete in 2-1 coordinate
τ_{lt}	applied shear stresses in the l-t coordinate of steel bars
ρ	steel ratio
ρ_l, ρ_t	longitudinal and transverse steel ratios respectively
ρ_{total}	$\rho_l + \rho_t$; total steel ratio
$(v_{12})_{Shear}$	Hsu/Zhu ratio v_{12} used in the SMM
$(v_{12})_{Torsion}$	modified Hsu/Zhu ratio used in the SMMT for torsion
θ	angle of twist per unit length
θ_{cr}	cracking angle of twist per unit length
θ_i	terminal θ of the initial $T-\theta$ straight line
ζ	softened coefficient of concrete in compression

unified rational SMMT theory for both solid and hollow RC members [12,21]. This project extending the theory to hollow specimens proposed and used a new test apparatus and method, and near perfect agreement was found between the pre-cracking branches of the nine analytical and nine experimental torque-twist curves. The experiment thus validated (1) the test apparatus and the method's ability to accurately measure small twist angles before and around cracking and (2) the SMMT theory's perfect accuracy in predicting the pre-cracking branch of a torque-twist curve. In other words, experiments have confirmed