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Shear strength of concrete members without transverse reinforcement: A mechanical approach to consistently account for size and strain effects

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ABSTRACT

Many theories and empirical formulae have been proposed to estimate the shear strength of reinforced concrete members without transverse reinforcement. It can be noted that these approaches differ not only in the resulting design expressions, but also on the governing parameters and on the interpretation of the failure mechanisms and governing shear-transfer actions. Also, no general consensus is yet available on the role that size and strain effects exhibit on the shear strength and how should they be accounted. This paper reviews the various potential shear-transfer actions in reinforced concrete beams with rectangular cross-section and discusses on their role, governing parameters and the influences that the size and level of deformation may exhibit on them. This is performed by means of an analytical integration of the stresses developed at the critical shear crack and accounting for the member kinematics. The results according to this analysis are discussed, leading to a number of conclusions. Finally, the resulting shear strength criteria are compared and related to the Critical Shear Crack Theory. This comparison shows the latter to be physically consistent, accounting for the governing mechanical parameters and leading to a smooth transition between limit analysis and Linear Elastic Fracture Mechanics in agreement to the size-effect law provided by Bažant et al.

1. Introduction

Design for shear of one- and two-way slabs without transverse reinforcement has been a topic where significant efforts have been devoted in the past. For beams and girders with stirrups, consistent equilibrium-based models as strut-and-tie models or stress fields can be applied [1]. However, with reference to the shear strength of beams and slabs without transverse reinforcement, no general agreement on the parameters and phenomena governing shear strength is yet found in the scientific community. This lack of agreement is also reflected in codes of practice, whose provisions for shear design are often based on empirical formulas [2,3]. Some approaches based on mechanical models consider a given shear transfer action as governing, neglecting the influence of the others. For instance, for one-way slabs without transverse reinforcement, shear carried by the compression chord is identified as the most significant parameter influencing the shear strength by Zararis [4]. On the contrary, aggregate interlocking can be considered as the governing shear transfer action explaining shear strength according to the compression field theory and its derivatives [5,6]. Also, Yang [7] acknowledges the role of aggregate interlock, whose failure is triggered by the development of a delamination crack at the level of the flexural reinforcement. Other approaches deal with the problem of shear strength in beams without transverse reinforcement on the basis of the tensile strength after cracking (including the presence of fibres in the cement matrix [8]) or based on fracture mechanics concepts [9,10]. Some interesting research lines have also been developed based on the upper-bound theorem of limit analysis with some modifications accounting for the presence of concrete cracking [11,12]. Finally, other approaches account for various potential shear-transfer actions. This is for instance the approach of Tue et al. [13] and Marí et al. [14] (where the role of the compression chord is nevertheless normally dominant) or the Critical Shear Crack Theory [15,16] (where the development of a critical shear crack limits the capacity of the shear-transfer actions). It is noticeable that, although different models account for different governing shear-carrying actions and for the strain and size effects in different manners, the final design expressions account for similar parameters with similar influences and, in most cases, fit in a similar manner when compared to available datasets.

An attempt to understand the role of the various potential shear transfer actions has recently been presented by Campana et al.

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Nomenclature

G_F	fracture energy
V	shear force
V_R	shear strength
V_{pl}	flexural strength
a	shear span
b	width of beam
b_{ef}	effective width in tension
c	thickness of compression zone
c_b	concrete cover
d	effective depth (distance from the centroid of the flexural reinforcement to the outermost compressed fibre)
d_B	depth of critical shear crack
d_b	bar diameter
d_g	maximum aggregate size
d_{g0}	reference aggregate size
d_n	dimension parameter
f_{ct}	tensile strength of concrete
f_c	concrete compressive strength measured in cylinder
f_{cef}	effective concrete compressive strength
k_i	coefficients
ℓ	lengths
s_b	bar spacing
w	crack width
w_{cr}	maximum crack width for which tensile stresses are transferred after concrete cracking
w_{li}	maximum crack width for which aggregate interlock stresses are transferred after concrete cracking
α	constant
β	constant; angle of compression strut
δ	crack slip
γ_i	shear span-to-effective depth ratio
θ	angle of inclined crack
ε	reference strain
σ	normal stress
τ	shear stress
σ_0	maximum normal stress transferred by aggregate interlocking
τ_0	Maximum shear stress transferred by aggregate interlocking
ψ	Rotation
η,ξ	Variables for integration

[17]. This investigation showed that different crack patterns may develop in similar reinforced concrete members and that their associated kinematics at failure (relative displacement of the lips of the critical shear crack) may also be very different. This holds true even for constant mechanical and geometrical parameters. Accounting for measured shapes and kinematics obtained by specific testing and by using a number of advanced constitutive models for aggregate interlock, residual tensile strength, doweling action and stirrup contribution, the contribution of each shear-transfer action to the total strength was estimated numerically. It was found that the governing shear transfer actions may be very different from one member to another. This dependency was mostly governed by the cracking pattern and its associated kinematics at failure, despite the fact that the total shear strength (sum of the various shear transfer actions) may be similar.

Other than the role attributed to the shear-transfer actions, different considerations are usually performed on the influence that size and strain effects have on shear strength. The size effect is defined as the reduction on the unitary (normalized) shear strength for geometrically identical specimens but with increasing size, refer to Fig. 1a. As stated by Bažant et al. [9,10], this reduction should follow a size-effect law, with a transition between a yield criterion for small sizes (without any size effect) and the behaviour predicted by Linear Elastic Fracture Mechanics (LEFM) for large sizes (strength reduction governed by $d^{-0.5}$). In addition, it has also been experimentally observed that specimens are sensitive to a strain effect [6,15], with decreasing unitary shear strength for geometrically identical specimens but subjected to higher levels of deformation (Fig. 1b). In many cases, both effects are considered by means of empirical coefficients, by introducing a size-effect factor (depending on the depth [9,10] or on shear span length [4,14]) and by relating the shear strength to the level of deformation (for instance as a function of the flexural reinforcement ratio or axial load [13,3]). Some design codes, however, neglect these aspects, at least in their most simplified design formulations [2].

In this paper, the contribution of the various shear-transfer actions to the shear strength and how they are influenced by the size and level of deformation of the member is investigated. This is performed by means of an analytical approach, accounting for their activation based on the shape of the shear crack and its kinematics and by using some fundamental constitutive models providing the stresses along the critical shear crack. By integration of the stresses at the critical shear crack, the contribution of each shear-transfer action is determined as well as its governing parameters. This allows obtaining eventually a failure criterion for shear design as well as to investigate on the influence of size and strain effects on the shear response. The results show that the contributions of all shear-transfer actions decrease for increasing openings of the critical shear crack and that their decay follows a similar trend. These results are finally related to the failure criterion proposed by the Critical Shear Crack Theory [15]. The works of this paper allow justifying on a rational basis its failure criterion (shape and influence of the various mechanical parameters considered by the theory). This criterion is observed to be consistent with the integration of stresses performed for the various critical shear crack shapes and kinematics investigated, thus validating the fundamental hypotheses of this theory. In addition, it is also shown that the theory is consistent with the strain effects and particularly with the size-effect law, providing naturally (without the need of considering any specific parameter) a smooth transition between a yield criterion and LEFM depending on the size of the member.

2. Shear-transfer actions in RC

After cracking due to bending, shear can be transferred in reinforced concrete members by a number of potential actions, whose activation depends much on the shape and kinematics of the critical crack leading to failure [17,15]. A summary of these actions is presented below (refer to Fig. 2):

- Cantilever action (Fig. 2a). The possibility of transferring shear by means of the concrete in between two flexural cracks (acting as a cantilever beam or "tooth" linking the tension and compression chords) was already observed by Kani [18]. At the location of the crack, shear is carried by the inclined compression chord. The strength of this action is limited by the development of the vertical flexural crack into a quasi-horizontal crack, which disables the capacity of the tension tie of the tooth [15].