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# Geometric nonlinear static and dynamic analysis of guyed towers using fully nonlinear element formulations

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ABSTRACT

The nonlinear responses of guyed towers under static loads and under dynamic loads are investigated by using the developed nonlinear Finite Element Method (FEM). Through the use of the developed fully nonlinear element formulations for structural components such as trusses, cables, and beams, it is possible to capture the complete geometric nonlinear response with the assistance of iterative algorithms such as Crisfield's method and the Newmark beta method. One 50 ft high guyed tower subjected to a constant top-concentrated load and one 328 ft guyed tower subjected to an EI Centro earthquake are used as demonstration. The results of the static analysis are compared with the results from SAP2000, ANSYS, and a linear equivalent column method. The results of the SAP2000, ANSYS, and the developed nonlinear FEM algorithm match well when the deflection is relatively small. In the Linear Equivalent Method, the equivalent spring-supported beam method provides the most accurate estimation when the deflection is relatively small. The complete time history of the guyed tower is also obtained for the nonlinear dynamic analysis.



## 1. Introduction

Guyed towers are one of the most efficient structures that can use a relatively small amount of materials to sustain relative large loads at significant heights. Therefore, guyed towers are often used as communication structures such as transmission, cellular, and antenna radio towers. It is composed of the mast and cables, which combines the advantage of combining a stiff structure (the mast) with a flexible structure (the cables). The vertical loads are transferred by the relatively stiff mast to the foundation, and the lateral movement of the mast is constrained by the flexible cables. The guy clusters are located at different heights along the mast and there can be as much as nine guy clusters in a high guyed tower. The mast base can be pinned or clamped to the ground.

The analysis and design of guyed towers are explored in detail in a number of documents and building codes, such as [1–4]. Some efficient ways to assess the responses of guyed towers are available in the literature [5–8]. Madugula [9] and Smith [10] studied the responses of guyed towers subjected to wind loads, whereas Irvine [11] has expanded the simplified dynamic analysis models under wind loads by using modal decomposition. Gerstoft and Davenport [12] has devised separate the guyed mast into high-frequency and low-frequency region using patching loads and dynamic magnification factor. Using a stiffness generation procedure, Desai et al. [13] analyzed a guyed tower subjected to a head load. Desai and Punde [14] proposed a nine-degree-of-freedom cable model applicable to cable-stayed structures subjected to gusty wind. Meshmesha et al. [15] developed empirical methods and formulas to estimate the seismic responses of guyed towers, and, a number of other studies [16,7,17,18] have addressed the same issue.

Various approaches to obtain the guyed towers' complex structural response have also been used by researchers. Ekhande and Madugula [19] conducted three dimensional guyed towers's geometric nonlinear analysis using linear Lagrangian coordinate framework. Kewaisy [20] adopted the combination of finite difference method and finite element method within total Lagrangian coordinate framework to analyze the dynamic response of guyed tower under wind induced forces. Amiri and McClure [21] used finite element computer program ADINA to analyzed the seismic response of guyed tower under three types of earthquakes. Jorge and Marta [22] employed finite element software ALGOR to model and study guyed towers' dynamic response.

However, compared to the state-of-the-art analysis and design techniques used for buildings and bridges, the corresponding techniques for guyed towers with large deformation are

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underdeveloped. When the guyed towers undergo large deformation, the global stiffness of the guyed towers has been significantly changed and the response becomes very different with the guyed towers without large deformation. The cables behave in a highly nonlinear way, the response of which is hard to predict using linear cable theories. The mast and cluster of cables also interact, which further increases the complexity of the analysis.

The objective of this paper is to present a developed nonlinear FEM that can fully accommodate the highly nonlinear behavior of the cables and the mast whether subjected to a static load or to a dynamic load. The FEM element formulation capable of simulating the large deformation is introduced first for the truss, cable, and beam elements. Then the iterative algorithm for the nonlinear static and dynamic analysis is presented. Finally, two cases are studied to illustrate the ability of the developed FEM system to predict both the nonlinear static response and the dynamic response of the guyed tower.

## 2. Nonlinear FEM element formulations

To capture the complex nonlinear behavior of the structure, fully nonlinear finite element formulations for truss elements, cable elements, and beam elements [23] have been validated with experimental study and are adopted.

### 2.1. Fully nonlinear FEM truss elements

The undeformed length of the truss element (Fig. 1) is

$$l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \tag{1}$$

where $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ are the position coordinates of points $P_1$ and $P_2$, respectively. Assume $(u_1, v_1, w_1)$ and $(u_2, v_2, w_2)$ are the deformation coordinates of points $P_1$ and $P_2$, respectively, such that the total Lagrange strain $\varepsilon$ along the deformed axis becomes

$$\varepsilon = \frac{\sqrt{(u_2 - u_1 + x_2 - x_1)^2 + (v_2 - v_1 + y_2 - y_1)^2 + (w_2 - w_1 + z_2 - z_1)^2} - l}{l}. \tag{2}$$

From the variation of elastic energy, Pai [23] found that the product of the stiffness matrix and the displacement vector is

$$[k]\{u\} = AlE\varepsilon\{\phi\} \tag{3}$$

where $E$ is the Young's modulus, $A$ is the cross-section area, $l$ is the length of the truss element, and $\{\phi\} = \left\{\frac{\partial\varepsilon}{\partial u_1}, \frac{\partial\varepsilon}{\partial v_1}, \frac{\partial\varepsilon}{\partial w_1}, \frac{\partial\varepsilon}{\partial u_2}, \frac{\partial\varepsilon}{\partial v_2}, \frac{\partial\varepsilon}{\partial w_2}\right\}^T$.

The tangent stiffness matrix can be obtained by taking the differentiate of Eq. (3) with respect to the displacement vector

$$[\breve{k}] = \frac{\partial([k]\{u\})}{\partial\{u\}} = EAl\{\phi\}\frac{\partial\varepsilon}{\partial\{u\}} + EAl\varepsilon\frac{\partial\{\phi\}}{\partial\{u\}} = EAl\{\phi\}\{\phi\}^T + EAl\varepsilon\frac{\partial^2\varepsilon}{\partial\{u\}^2} \tag{4}$$

The mass matrix is the same as that of the linear truss element in [24],

$$[m] = \frac{\rho Al}{6}\begin{bmatrix} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{bmatrix} \tag{5}$$

where $\rho$ is the mass density. Parametric study using this truss element can be found in [25].

### 2.2. Fully nonlinear FEM cable element

It is assumed that the static load is applied before the dynamic loads are applied. From Fig. 2, the axial strain [23] is

$$\varepsilon = \sqrt{(\alpha_1' + u_1')^2 + (\alpha_2' + u_2')^2 + (\alpha_3' + u_3')^2} - 1 \tag{6}$$

where $\{\alpha_1, \alpha_2, \alpha_3\}$ is the deformation vector under a static load, $\{u_1, u_2, u_3\}$ is the deformation vector under a dynamic load, and $(\,)' = \frac{\partial(\,)}{\partial s}$. From the variation of energy, it can be found that the product of the stiffness matrix and the displacement vector is

$$[k]\{u\} = \int_0^l EA_0[1 - \upsilon\varepsilon]^2\varepsilon[D]^T\{\phi\}ds \tag{7}$$

where the initial strain caused by the prestress in the bar is $e_0$, the Poisson's ratio is $\upsilon$, the original cross-section area is $A_0$, and the Young's modulus is $E$,

$$\{\phi\} = \left\{\frac{\alpha_1' + u_1'}{1+\varepsilon}, \frac{\alpha_2' + u_2'}{1+\varepsilon}, \frac{\alpha_3' + u_3'}{1+\varepsilon}\right\}, \quad \text{and}$$

$$[D] = \begin{bmatrix} -\frac{1}{l} & 0 & 0 & \frac{1}{l} & 0 & 0 \\ 0 & -\frac{1}{l} & 0 & 0 & \frac{1}{l} & 0 \\ 0 & 0 & -\frac{1}{l} & 0 & 0 & \frac{1}{l} \end{bmatrix}.$$

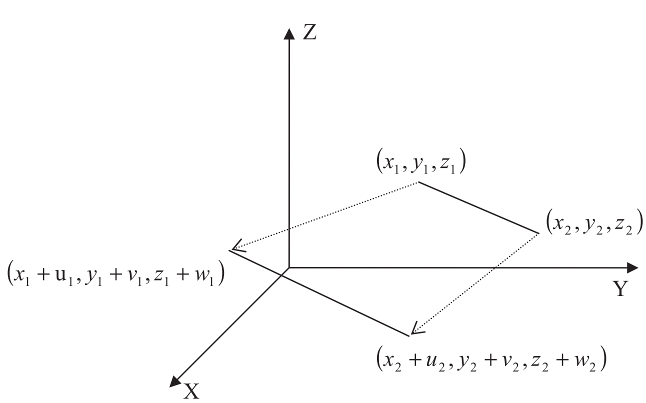


**Fig. 1.** Undeformed and deformed geometry of a truss element.

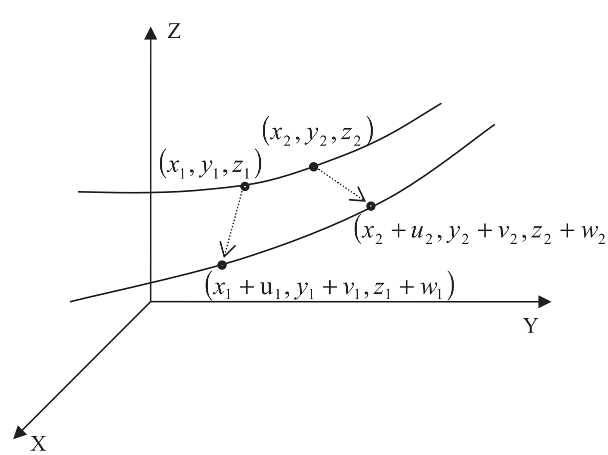


**Fig. 2.** Undeformed and deformed geometry of a cable element.