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Nonlinear finite element analysis of elastic water storage tanks

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ABSTRACT

The present paper deals with the nonlinear finite element analysis of elastic water tanks. Both fluid and tank walls are discretized and modeled by two dimensional eight-node isoperimetric elements. In the governing equations, displacement for tank walls and pressure for the fluid domain are considered as independent nodal variables. The nonlinear term for the convective acceleration in Navier–Stokes equations is incorporated in the present analysis. The nonlinearity effects of the fluid are studied considering excitations both harmonic of various frequencies and random. The hydrodynamic pressure on tank wall is presented both from linear and nonlinear analysis for a comparison. The results show that the convective nonlinearity increases the magnitude of hydrodynamic pressure to a considerable amount when the excitation frequency approaches to the fundamental frequency of the water tank. The magnitude of hydrodynamic pressure in nonlinear analysis is quite large compare to that in linear analysis when the distance between the two vertical walls are relatively closer. However, the increase in pressure is insignificant when the tank is vibrated with a frequency higher than the frequency of the fluid domain. The seismic analysis results show that the distribution of nonlinear hydrodynamic pressure is almost similar to the linear pressure due to ground excitation.

1. Introduction

Tanks are commonly used to store water and various fluids in the oil industry. Damages in tanks may cause a loss of liquid content, which could result in economic damage, as well as in long-term contamination of soil for tanks resting on soil. For the design of an earthquake-resistant water tank, it is important to use a rational and reliable dynamic analysis procedure. The analysis procedure should be capable of evaluating the dynamic deformations and hydrodynamic pressure on tank wall under probable ground motion.

For simplification of the analytical procedures, the tank walls are generally considered to be rigid. Housner [1] presented analytical solutions for rectangular and cylindrical rigid tanks under horizontal ground excitations. An analytical or a closed-form solution cannot account for the arbitrary geometry of the tank. This problem can be efficiently tackled with finite element technique. For finite element modeling of fluid, different variables such as displacement, pressure, velocity potential etc. are considered. Fenves and Vargas [2] considered velocity and pressure as nodal variables to represent the governing equations of fluid. Zienkiewicz et al. [3] represented the fluid domain in terms of a displacement potential. Pelecanoset et al. [4] considered displacement as nodal variable to model the reservoir. Bouaanani and Lu [5] considered the velocity potential be the nodal variable for the analysis of reservoir. Maity and Bhattacharyya [6] and Gogoi and Maity [7] considered pressure to be nodal degree of freedom in the finite element modeling of the fluid.

Tung [8] studied the hydrodynamic pressure distribution on a rigid, submerged, cylindrical storage tank subjected to horizontal base excitation. The author considered fluid to be incompressible. Williams and Moubayed [9] calculated hydrodynamic pressure on rigid tank walls by boundary integral method due to horizontal and vertical excitations considering fluid to be linearly compressible. The effects of a bottom-mounted rectangular block on the sloshing characteristics of the fluid in rectangular tanks were investigated by Choun and Yun [10] using the linear water wave theory. The hydrodynamic pressure exerted on the tank wall and on the mounted block is changed significantly as the block becomes large and moves toward the wall. Consequent experimental study by Panigrahy el al. [11] showed that the pressure exerted on the walls varies in a similar nature as that of the applied excitation and the pressure at the wall showed a considerable fluctuation near the free surface of the liquid compared to the deeper surfaces in the tank. A time-independent finite difference method was developed by Hua et al. [12] to simulate fluid sloshing and sloshing induced hydrodynamic pressure on tank walls. The

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authors showed that the horizontal hydrodynamic force due to sloshing waves acting on the mid-section of the wall of the tank is dominated by the added mass effect when the excitation frequency is larger than four times the lowest natural frequency of the tank. They also concluded that sloshing waves plays a key effect on the horizontal sloshing-induced force if the excitation frequency is less than four times the fundamental frequency of the tank. However, all the above numerical simulations are carried out considering tank walls to be rigid. The deformations, stresses and hydrodynamic pressure will be more realistic if the elastic properties of tank walls and wall–fluid interaction are considered appropriately.

Haroun and Tayel [13] analyzed the earthquake response of cylindrical liquid storage tanks under vertical excitations and showed that structures are more susceptible to the increase in hoop for vertical excitation. Similar study on cylindrical liquid storage tank was carried out by Veletsos and Tang [14] and Haroun and Abou [15] considering tank walls to be flexible. Kianoush and Chen [16] used finite element technique to obtain hydrodynamic pressure on a rectangular tank due to vertical excitation. Park et al. [17] used the coupled boundary-finite element method to study the response of similar rectangular tanks. Chen and Kianoush [18] calculated hydrodynamic pressure on rectangular tank wall by lumped mass approach as well as consistent mass approach and their results showed that the consistent mass approach reduces the response of liquid containing structures as compared to that of lumped mass approach. Moslemi and Kianoush [19] studied the influences of sloshing of free surface, tank wall flexibility, vertical ground acceleration, tank aspect ratio and base fixity on hydrodynamic pressure. The authors also compared the results with those calculated from well-proved analytical methods and concluded that current design procedure in estimating the hydrodynamic pressure was too conservative. The estimated hydrodynamic pressure will be more realistic particularly for strong ground motion when the fluid nonlinearity is considered properly.

A nonlinear liquid sloshing inside a partially filled rectangular tank was investigated by Celebi and Akyildiz [20]. Authors used volume of fluid technique to track the free surface and the model solved the complete Navier–Stokes equations in primitive variables by use of the finite difference approximations. Similar nonlinear sloshing for cylindrical water tank was determined by Barrios et al. [21]. Virella et al. [22] used linear and nonlinear wave theories to determine the linear and nonlinear hydrodynamic pressure on rigid tank walls using the finite element software ABAQUS. The pressure and displacement were considered as unknown variables for linear and nonlinear model respectively. A plane strain formulation with large displacements was used for nonlinear modeling and their study showed that the nonlinearity does not have significant effects on the natural sloshing periods. The linear wave theory conservatively estimated the magnitude of the pressure distribution, whereas larger resultant pressure is obtained if nonlinear theory is considered.

The exhaustive literature review reveals that no study has been carried out on the nonlinear behavior of water storage tanks considering tank flexibility. The accurate behavior of the water tank can be achieved if the coupled fluid–structure interaction effects are taken into consideration in the analysis. Generally, the displacement based finite element method is used to model the tank-water coupled system. However, many researchers solved Navier–Stokes equation with pressure term as variable parameter only in their modeling. In the present study, the nonlinear convective terms in the Navier–Stokes equation is taken into account in an iterative manner while solving the fluid equation. Water in the tank is assumed to be compressible. A pressure based finite element model is developed to obtain nonlinear response of fluid domain. The tank wall is modeled in two dimensions and coupled with the fluid equation. A computer code in MATLAB environment has been developed to obtain hydrodynamic pressure, displacement and stresses on tank wall. The study is carried out for different amplitudes and frequencies of harmonic excitation and earthquake excitation to investigate the degree of nonlinear effects. The effects of fluid nonlinearity are also studied for different lengths of the tank.

2. Theoretical formulations for fluid

The state of stress for a Newtonian fluid is defined by an isotropic tensor as [23]

$$T_{ij} = -p\delta_{ij} + T'_{ij} \tag{1}$$

where T_{ij} is total stress, T'_{ij} is viscous stress tensor which depends only on the rate of deformation in such a way that the value becomes zero when the fluid is under rigid body motion or rest. The variable p is defined as hydrodynamic pressure whose value is independent explicitly on the rate of deformation and δ_{ij} is kronecker delta. For isotropic linear elastic material, the most general form of T'_{ij} is

$$T'_{ij} = \lambda\Delta\delta_{ij} + 2\mu D_{ij} \tag{2}$$

Where μ and λ are two material constants. μ is known as first coefficient of viscosity or viscosity and $(\lambda + 2\mu/3)$ is second coefficient of viscosity or bulk viscosity. D_{ij} is the rate of deformation tensor and is expressed as

$$D_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial y_j} + \frac{\partial v_j}{\partial x_i}\right) \quad \text{and} \quad \Delta = D_{11} + D_{22} + D_{33} \tag{3}$$

Thus, the total stress tensor becomes

$$T_{ij} = -p\delta_{ij} + \lambda\Delta\delta_{ij} + 2\mu D_{ij} \tag{4}$$

For compressible fluid, bulk viscosity $(\lambda + 2\mu/3)$ is zero. Thus, Eq. (4) becomes

$$T_{ij} = -p\delta_{ij} - \frac{2\mu}{3}\Delta\delta_{ij} + 2\mu D_{ij} \tag{5}$$

If the viscosity of fluid is neglected, Eq. (5) becomes

$$T_{ij} = -p\delta_{ij} \tag{6}$$

Generalized Navier–Stokes equations of motion are given by

$$\rho\left(\frac{\partial v_i}{\partial t} + v_j\frac{\partial v_i}{\partial x_j}\right) = \frac{\partial T_{ij}}{\partial x_j} + \rho B_i \tag{7}$$

where B_i is the body force and ρ is the mass density of fluid. Substituting Eq. (6) in Eq. (7) the following relations are obtained.

$$\rho\left(\frac{\partial v_i}{\partial t} + v_j\frac{\partial v_i}{\partial x_j}\right) = \rho B_i - \frac{\partial p}{\partial x_i} \tag{8}$$

If u and v are the velocity components along x and y axes respectively and f_x and f_y are body forces along x and y direction respectively, the equation of motion may be written as

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = f_x \tag{9}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = f_y \tag{10}$$

Neglecting the body forces, Eqs. (9) and (10) become

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 0 \tag{11}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 0 \tag{12}$$