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An efficient algorithm for simultaneous identification of time-varying structural parameters and unknown excitations of a building structure

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ABSTRACT

When a structure is being damaged under unknown excitations, structural parameters of the damaged elements are actually varying with time. Hence, the simultaneous identification of time-varying structural parameters and unknown excitations is an important task in structural health monitoring. Although some analytical methods for such identifications are available in the literature, they are complicated, time-consuming, or restrained by special requirements. This paper presents an efficient algorithm for identifying time-varying structural parameters of a building structure under unknown excitations. By projecting on to the column space of influence matrix of unknown excitations, the observation equation of the structural system with unknown excitations is transformed from a multiple linear regression equation to a simple linear regression equation. By further introducing a time-varying correction factor matrix, an analytical recursive least-squares estimation algorithm is developed for identifying unknown excitations and time-varying structural parameters such as stiffness and damping. The feasibility and accuracy of the proposed algorithm are finally demonstrated through numerical examples and comparison with the existing methods. The results clearly exhibit that the proposed algorithm can simultaneously identify unknown excitations and time-varying structural parameters efficiently and accurately.

1. Introduction

Structural parameters of damaged elements will vary with time when a structure is being damaged under unknown excitations. The simultaneous identification of time-varying structural parameters and unknown excitations when damage occurs is therefore an important task in structural health monitoring. A great deal of studies has been conducted for structural parameter identification and damage detection in the last few decades in either the frequency domain [1–5], time domain [6–10] or time–frequency domain [11–16]. Nevertheless, most of these studies identify the structural parameters prior to and after the event and the constant damage is then determined by a comparison of the identified structural parameters, which is difficult to realize on-line damage detection.

To catch the structural damage variation on-line, a number of methods have been proposed for tracking variations of structural modal parameters and physical parameters. These methods include Hilbert transform-based methods, wavelet transform-based methods, neural network-based methods and adaptive tracking-based methods. As an efficient signal processing method, the Hilbert transform method has been applied to structural identification for time-varying systems. Shi et al. [17] proposed an algorithm based on the Hilbert transform and the empirical mode decomposition with forced vibration response data for the identification of linear time-varying multiple degrees-of-freedom systems. Wang et al. [18] further developed a new recursive Hilbert transform method for time-varying property identification of large-scale shear-type buildings, in which an observer technique and a branch-and-bound technique were used. Feldman [19] demonstrated a Hilbert transform method to extract instantaneous dynamic structure characteristics in the time domain and identify time-varying mechanical vibration systems under free and forced vibration regimes. Similar to the Hilbert transform method, the wavelet transform has successfully been used in the identification of time-varying systems as an advanced time–frequency windowing technique. Ghanem and Romeo [20] presented a discrete wavelet identification approach for analysis of time-varying structures. The algorithm was associated with a differential equation model

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relating input and output responses using the wavelet Galerkin approach. Basu et al. [21] developed an online identification of variation of stiffness in structural systems based on wavelet analysis. By modifying the Littlewood–Paley wavelet, they used time–frequency characteristics of the wavelets in the online identification. Su et al. [22] introduced a continuous wavelet transform-based approach to identify the instantaneous modal parameters of a linear time-varying structure and its substructures in the frequency ranges of interest. The neural network is another attractive technique which can be used to identify variations of structural parameters. Qian and Mita [23] proposed an acceleration-based method using artificial neural network emulators to evaluate building structures under earthquake excitation. Ahmed-Ali et al. [24] presented a new scheme for time-varying parameter identification using sliding-neural network observer, in which the Lyapunov arguments and sliding mode theories were used for achieving the convergence of the proposed algorithm. Recently, various methodologies have been developed based on the adaptive tracking technique used to track time-varying structural parameters. Smyth et al. [25] introduced a constant forgetting factor to find structural parametric variations on-line, but the method is sensitive to measurement noise. To obtain a better tracking capability, a variable trace method was then proposed by Lin et al. [26] to adjust the adaptation gain matrix and capture parameter changes as time moves forward. Paleologu et al. [27] also introduced a variable forgetting factor to obtain good performance of time-varying parameter identification. Yang and Lin [28] used an adaptive tracking technique to detect damage on-line based on measurement vibration data. However, most of these methods mentioned above require that all the external excitations be known or measured.

In view of practice, it is difficult to measure all external excitations accurately because of the measurement technique and cost. To identify structural parameters without input information, several studies have been performed in the time domain, such as random decrement technique [29], free vibration response analysis [30], statistical average algorithm [31], hybrid identification algorithm [7], substructure approach [32], genetic algorithms-based method [33], and sensitivity-based method [34,35]. Nevertheless, these time domain techniques have special requirements such as zero mean input excitations or free-decay responses. To abolish these restrictions on input and output, least-squares estimation methods and extended Kalman filter methods [10,36–38] have been utilized for system identification without any input information. Wang and Haldar [39] proposed an iterative algorithm for identifying constant structural parameters based on the traditional least-squares estimation method. Because this method does not require input information, there is no limit for the type of external excitations. Wang and Haldar [40] then combined the proposed method with the extended Kalman filter for improving the accuracy of system identification. Ling and Haldar [41] further presented a modified iterative least-squares algorithm and verified it with several numerical examples. Unlike the iterative algorithms mentioned above, Yang et al. [42–44] presented an analytical recursive solution based on the least-squares estimation method by placing the unknown input excitation into an extended unknown vector, and then they combined the proposed method with an adaptive tracking technique for the case in which the structural parameters are varying.

This paper presents an efficient algorithm for identifying time-varying structural parameters of a building structure with unknown excitations based on the least-squares estimation method. To utilize the least-squares estimation method directly, the observation equation of the structural system with unknown excitations is transformed from a multiple linear regression equation to a simple linear regression equation by projecting on to the column space of influence matrix of unknown excitations. By further introducing a time-varying correction factor matrix determined by measurement data, an analytical recursive least-squares estimation algorithm is then developed for identifying unknown excitations and time-varying structural parameters such as stiffness and damping. The feasibility of the proposed algorithm is finally demonstrated in terms of an example shear building, in which different types of structural damage, structural damping, and excitations are considered with the measurement noise included. The results are also compared with those from the existing algorithm to examine the efficiency and accuracy of the proposed algorithm.

2. Proposed algorithm

2.1. Equation of motion

In this study, the structural mass matrix $\boldsymbol{M}$ is assumed to be known for simplicity of presentation although it is not absolutely necessary. Other structural parameters, such as damping and stiffness coefficients, are time-varying as structural damage occurs with time. For generality, external excitations are separated into two parts: measured excitations and unknown excitations. Consequently, the second-order differential equation of motion of a building structure with n DOF is given by

$$\boldsymbol{M}\ddot{\boldsymbol{X}}(t) + \boldsymbol{C}(t)\dot{\boldsymbol{X}}(t) + \boldsymbol{K}(t)\boldsymbol{X}(t) = \boldsymbol{\varphi}^*\boldsymbol{f}^*(t) + \boldsymbol{\varphi}\boldsymbol{f}(t) \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}(t)$ and $\boldsymbol{K}(t)$ represent the mass, damping and stiffness matrices of the building structure, respectively; $\ddot{\boldsymbol{X}}(t)$, $\dot{\boldsymbol{X}}(t)$ and $\boldsymbol{X}(t)$ are the $n \times 1$ structural acceleration, velocity and displacement response vectors, respectively; $\boldsymbol{f}^*(t)$ is the $r \times 1$ measured excitation vector with the influence matrix $\boldsymbol{\varphi}^*$ $(n \times r)$; and $\boldsymbol{f}(t)$ is the $s \times 1$ unknown excitation vector with the influence matrix $\boldsymbol{\varphi}(n \times s)$.

2.2. Multiple linear regression equation

Suppose that $\boldsymbol{Z}$ is an $m \times 1$ unknown time-varying structural parameter vector, which include structural stiffness and damping. The unknown time-varying structural parameter vector at time $t = k\Delta t$ is denoted as $\boldsymbol{Z}_k$, in which Δt is the sampling interval.

As presented by Yang et al. [44], the observation (measurement) equation associated with Eq. (1) at time $t = k\Delta t$ can be described as

$$\boldsymbol{y}_k = \boldsymbol{H}_k\boldsymbol{Z}_k - \boldsymbol{\varphi}\boldsymbol{f}_k + \boldsymbol{v}_k \tag{2}$$

where $\boldsymbol{y}_k = -\boldsymbol{M}\ddot{\boldsymbol{X}}_k + \boldsymbol{\varphi}^*\boldsymbol{f}_k^*$ is a $n \times 1$ measurement vector which can be obtained by the measured structural acceleration responses $\ddot{\boldsymbol{X}}_k$ and the measured excitations $\boldsymbol{f}_k^*$; $\boldsymbol{H}_k$ is an $n \times m$ observation matrix composed of the measured structural velocity and displacement responses $\dot{\boldsymbol{X}}_k$ and $\boldsymbol{X}_k$; $\boldsymbol{v}_k$ represents a $n \times 1$ noise vector, taking into consideration the model uncertainty of the structure and the measurement noise. The noise vector can be assumed as a white noise with normal probability distribution. The subscript k represents the values of matrices or vectors at time $t = k\Delta t$.

There are two predictor variable vectors $\boldsymbol{Z}_k$ and $\boldsymbol{f}_k$ in Eq. (2). Therefore, Eq. (2) is a multiple linear regression equation. Furthermore, the elements in the predictor variable vectors are time-varying. The traditional least-squares estimation method for solving the simple linear regression equation to find constant structural parameters cannot be directly applied to Eq. (2).

2.3. Transformation to simple linear regression equation

To transform the multiple linear regression equation to a simple linear regression equation, Yang et al. [44] moved the unknown excitation vector $\boldsymbol{f}_k$ into the unknown structural parameter vector