Engineering Structures 90 (2015) 38–47

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Interaction diagrams of reinforced concrete sections in fire: A yield design approach

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ARTICLE INFO

Article history:
Received 23 December 2013
Revised 10 February 2015
Accepted 11 February 2015
Available online 2 March 2015

Keywords:
Reinforced concrete section
Interaction diagrams
Fire loading
Yield design approach

ABSTRACT

Based upon the yield design (or limit analysis) theory, a straightforward computational procedure allowing to determine the interaction diagrams relating to combined axial and bending loadings of reinforced concrete sections in fire, is presented in this paper. It is shown in particular how both the lower and upper bound methods, leading to the exact determination of such interaction diagrams, can be easily implemented, where the local degradation of the material strength capacities as a function of temperature increase represents a key feature of the analysis. Such theoretical predictions prove to be in quite good agreement with both Eurocode-based calculations at ambient temperature and available experimental results in the case of fire-loaded reinforced concrete beams.

1. Introduction

Being selected for several reasons among which their ability to provide a convenient graphical illustration of the ultimate axial-bending capacity of structural members, interaction diagrams represent a widely used practical tool for the design of reinforced concrete structures. As described for instance in Eurocode 2-Part 1-1 [1], such failure curves can be classically established by considering all admissible stress distributions across the depth of the structural member, in the framework of ultimate limit state design (ULSD). More precisely, three characteristics points, usually referred to as "pivots", are defined corresponding to different strain limitations relating to reinforcing steel in tension and concrete in compression. These strain limitations correspond to conventional values which depend solely on material characteristics.

However, such a calculation procedure based on strain limitations becomes much more complex when attempting to design reinforced concrete sections under thermal loading, since those strain limitations are now temperature dependent and fire-induced thermal loads cause material properties degradation. For this kind of problem, the 500 °C *isotherm method* advocated in Eurocode 2-Part 1-2 [2], remains the only available method so far, based on the very crude approximation that concrete is totally damaged for temperatures higher than 500 °C, while the residual concrete section retains its initial properties. This assumption implies that some of the reinforcing bars may fall outside the reduced cross-section. Furthermore, this simplified calculation based on such reduced sections may be non-conservative and therefore become highly questionable, notably in the case of small sections, which are much more temperature sensitive.

Several contributions have been dealing with the appropriate way for assessing the ultimate behavior of reinforced concrete members subjected to fire loading (see for example: Lie and Celikkol [3], Lie and Irwin [4], Dotreppe et al. [5], Meda et al. [6], Franssen and Dotreppe [7] or El-Fitiany and Youssef [8]). Two recently and independently published papers have more thoroughly addressed the issue of determining the interaction diagrams of heated reinforced concrete sections. Caldas et al. [9], for example, have developed a general method based on an algorithm for the construction of strength interaction diagrams for arbitrary-shaped reinforced concrete sections subjected to fire actions. The diagrams are obtained through the prescription of conventional ultimate strain values for concrete and steel, where fire-induced thermal strains are taken into account. The obtained results are in good agreement with those of Meda et al. [6]. On the other hand, Law and Gillie [10] have proposed an alternative method making use of the tangent stiffness matrix relating the incremental axial force and bending moment to the corresponding axial strain and

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http://dx.doi.org/10.1016/j.engstruct.2015.02.012
0141-0296/

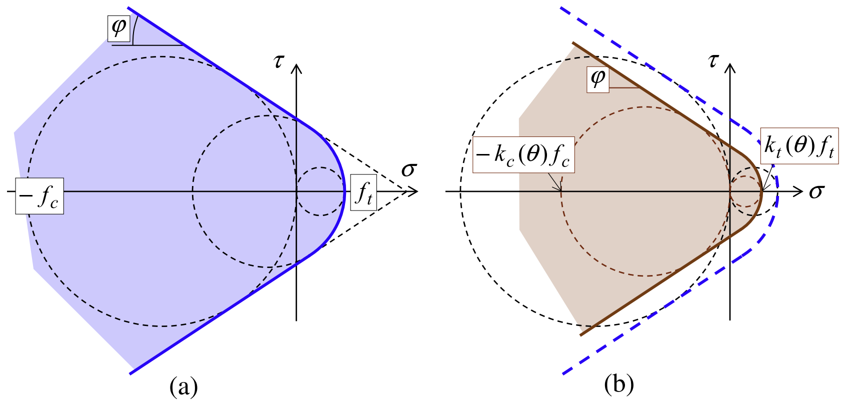

Fig. 1. Intrinsic curve of concrete at (a) ambient and (b) elevated temperature.

curvature increments. The point where this matrix becomes singular is searched by means of an effective search algorithm, the associated stress resultants being conventionally located on the interaction diagram. Even though these methods represent substantial improvements with respect to the previous ones, their implementation still remains rather complex in the context of engineering design.

The present contribution is based on yield design (or limit analysis) theory for deriving such interaction curves in a much more straightforward and efficient way than the previously mentioned design methods. Indeed, while the latter methods are based on conventional limitations concerning the strains experienced by concrete and reinforcing steel, the yield design approach, as applicable to various problems in the field of civil engineering [11–14], only requires that *stress* (and not *strain*) limitations be prescribed to the constituent materials in the form of a strength or failure criterion, with no reference to other mechanical characteristics, such as deformability properties. To put it simply, this theory is fundamentally based on verifying the compatibility between static equilibrium of a structure subjected to given loading conditions and strength requirements expressed through the materials failure criteria, without it being necessary to resort to tedious and sophisticated incremental computations. It therefore provides the perfectly suitable framework for performing ultimate limit state design of structures.

The application of this approach to the failure design of reinforced concrete beams subjected to normal force and bending moment solicitations at ambient temperature has been shown in [15,16], producing simple analytical easy-to-use formulations of the corresponding interaction diagrams. Quite recently, Bleyer and de Buhan [17] have extended the method to the determination of combined bi-axial bending interaction surfaces of composite beams, which can then be incorporated in the failure analysis and design of complex space frame structures under isothermal conditions.

The purpose of the present contribution is to generalize the yield design approach to the case of reinforced concrete sections subjected to a fire-induced temperature increase, in addition to classical mechanical loading conditions. The key feature of the analysis is the introduction of experimentally-based relationships linking the degradation of local material strength properties to the fire-induced temperature increase. The interaction diagram associated to a prescribed temperature profile is thus exactly determined from implementing both the lower bound static and upper bound kinematic methods. It is shown in particular that the temperature increase affects the strength properties of the reinforced concrete section, in the form of a quite significant reduction of the interaction diagram. The solution thus obtained is favorably compared with Eurocode-based predictions in the case of ambient temperature, as well as with available experimental results in fire conditions.

2. Material strength properties under fire

In this section, failure criteria will be defined for the different constituent materials (concrete and steel), first in the case of ambient temperature, then in order to incorporate the effects of high temperatures, under the assumption that spalling does not occur. Although the introduction of a specific strength condition for the reinforcement-concrete interface would not present major difficulties, perfect bonding between the reinforcing bars and the surrounding concrete will be provisionally assumed in the following.

2.1. Failure criterion for plain concrete

Following the approach proposed by Chen [18]; Salençon [11]; Averbuch and de Buhan [19] or quite recently Nielsen and Hoang [20], a modified Mohr–Coulomb failure condition will be adopted for ascertaining the strength properties of plain concrete. The expression of this criterion is the following:

$$f^c(\underline{\underline{\sigma}}) = \sup_{i,j=1,2,3} \{\sigma_i(1+\sin\varphi) - \sigma_j(1-\sin\varphi) - f_c(1-\sin\varphi); \sigma_i - f_t\} \leqslant 0 \tag{1}$$

where $\sigma_i, i = 1,2,3$, are the principal stress components, while f_t, f_c and φ are three material parameters, known respectively as the uniaxial tensile and compressive strengths, and the internal friction angle at ambient temperature. According to Nielsen and Hoang [20], this friction angle may be taken as approximately equal to 37° for ordinary concrete. A representation of this criterion in the Mohr plane in the form of an intrinsic curve is shown in Fig. 1(a).

An important assumption will be made from now on, as regards the effect of temperature on the strength parameters introduced in (1), namely that the friction angle will remain unaffected by the temperature increase; therefore, it will be conventionally kept constant and equal to $\varphi = 37°$. On the contrary, the tensile and compressive strengths will be reduced through the introduction of non-dimensional multiplicative factors:

$$f_t(\theta) = k_t(\theta) f_t \quad \text{and} \quad f_c(\theta) = k_c(\theta) f_c \tag{2}$$

where k_t and k_c are decreasing functions of the temperature increase θ, equal to one for $\theta = 20$ °C (ambient temperature). The corresponding modified intrinsic curve is displayed in Fig. 1(b).

2.2. Failure criteria for steel reinforcing bars

The reinforcing bars are modeled as linear (i.e. 1D) structural elements embedded in the concrete material described as a 3D continuum. Neglecting their shear and bending resistance, their