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Effect of load history on punching shear resistance of flat slabs

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ABSTRACT

The unloading of reinforced concrete slabs results in residual slab rotations and reloading to the same load results in irreversible rotation increases. Unloading and reloading (UR) cycles applied to non-strengthened and strengthened flat slabs may thus affect the punching resistance, which is rotation-dependent. A quintilinear moment–curvature relationship, which takes concrete softening and tension stiffening into account, combined with UR cycles, modeled as bilinear envelopes, is developed to predict residual slab rotations and irreversible rotation increases. A parametric study shows that the effect of UR cycles on the punching resistance of concrete is normally small, however, it may be significant if the slab is strengthened after unloading, particularly for thin and low-reinforced slabs, which exhibited plastic slab rotations before unloading. Prestressing of the strengthening system may reduce the residual slab rotations and thus limit or compensate the loss of punching resistance.

1. Introduction

In contrast to the bending resistance, the punching shear resistance of flat slabs depends on the slab rotation [1]. For slabs with interior columns, the punching resistance can be estimated with analytical formulas, which are based on the assumption of a rotation-symmetric slab cutout. Load–rotation curves can be derived for different, e.g. bilinear or quadrilinear, moment–curvature relationships [2]. Based on the Tension Chord Model (TCM) [3], the quadrilinear relationship takes a contribution of the concrete tensile stresses between the cracks into account. This leads to a decrease of the average steel stresses in the cracked zone and thus to the so-called “tension stiffening effect”, which may influence the punching resistance of slabs with low reinforcement ratios of the longitudinal reinforcement [2].

In addition to the consideration of concrete tensile stresses between cracks, the behavior of concrete in the crack itself was also investigated. Studies on cement pastes [4,5] showed that their crystalline structure can hinder or even arrest the growth of microcracks by means of interlocking fibers growing out from the cement grains. Thus the concrete tensile stresses do not abruptly drop to zero after tensile strength is reached, but still provide a fractional contribution up to a critical (fictitious) crack width, w_{cr}. This behavior is considered by the Fictitious Crack Model (FCM) [6,7] for instance, where one sharp crack of zero initial length is assumed, or by the blunt Crack Band Model (CBM) [8,9], which smears the crack over a fracture process zone of a certain width.

A Modified Sector Model (MSM) has been proposed by the authors [10], which takes into account the fact that the punching resistance also depends on the level of transverse shear loading. This model considers a quadrilinear moment–curvature relationship including the tension stiffening effect. Comparisons with experimental results indicated that slab rotations generally agree very well, but are slightly overestimated immediately after the cracking of the slab [10]. This overestimation was attributed to the tensile contribution in the fracture process zone, which had not been taken into account.

The unloading and reloading of slabs may further influence the deformation behavior and thus the punching resistance. Prior unloading of slabs may be required (e.g. by bracings) if existing slabs have to be strengthened against punching shear in order to activate the post-installed strengthening systems. For the uniaxial tension chord it was shown [11] that unloading and reloading (UR) cycles influence the deformation behavior by affecting the bond properties between the reinforcing steel and surrounding concrete. A residual slab rotation, ψ_{res}, was noticed after unloading of the slab [12]. After reloading up to the same load as previously applied, an irreversible increase of rotation, $\Delta\psi_{UR}$, was observed [13,14]. Some experiments even failed in punching before reaching the

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Nomenclature

A_c	concrete cross-sectional area
A_s, a_s	cross-sectional area of reinforcement (total, per unit width)
$A_{t,ef}$	effective tension area (tension chord)
B	side length of slab
E_c	concrete Young's modulus
E_s	steel Young's modulus
EI^{I}	uncracked flexural slab stiffness per unit width $= E_c h^3/12$
EI^{II}	cracked flexural slab stiffness per unit width
G	dead weight
G_F	specific fracture energy
L	initial length of beam element
M	bending moment
N	axial force
N_{cr}	cracking load
$N_{\sup}$, $N_{\inf}$	upper and lower bounds of axial force (UR cycle)
V	shear force
V_{flex}	flexural capacity of non-strengthened slab
V_{mod}	shear force (MSM), $= f(\psi)$
V_{Rc}	failure criterion (CSCT), $= f(\psi)$
$V_{R,crush}$	crushing resistance of concrete, $= f(\psi)$
V_{R0}	predicted punching shear resistance (non-strengthened slab) or portion of punching shear resistance borne by concrete
V_{R1}	predicted punching shear resistance (strengthened slab)
V_{skt}	shear force (CSCT), $= f(\psi)$
$V_{\sup}$, $V_{\inf}$	upper and lower bounds of shear force (UR cycle)
b	side length of column
d	effective depth
d_g	maximum aggregate size
d_m	average effective depth of both orthogonal directions
d_v	shear-resisting effective depth
f_c	(cylinder) concrete compressive strength
f_{ct}	concrete tensile strength
f_{sy}	yield strength of reinforcing steel
h	slab thickness
k_g	factor to take crack roughness into account
k_{sys}	factor to take performance of shear reinforcement system into account
k_τ	bond stress factor
m_{cr}	cracking moment per unit width
m_r, m_t	radial and tangential moments per unit width
m_R	bending resistance (average of both rebar directions)
$m_{\sup}$, $m_{\inf}$	upper and lower bounds of moment (UR cycle)
n	modular ratio $= E_s/E_c$
r	radius (from slab center)
r_0	radius of critical shear crack (CSCT) $= r_c + d_m$
r_c	radius of (equivalent) circular column
r_{cr}	radius of cracked zone
$\tilde{r}_{cr}$	radius of cracked zone at critical crack width w_{cr}
r_{crs}	radius of zone in which cracking is stabilized (at m_{cr})
$\tilde{r}_{crs}$	radius of zone in which cracking is stabilized (at $m_{cr} + \Delta m_{cr}$)
r_q	radius of load introduction at perimeter
r_s	(equivalent) radius of circular isolated slab element = slab radius
r_y	radius of yielded zone
s_{rm}	average crack spacing
u_0	control perimeter for punching shear resistance (distance $d_v/2$ to supported area)
w	crack width
w_{cr}	critical crack width where concrete tensile strength is exhausted
x	horizontal coordinate (axial direction), depth of compression zone
y	horizontal coordinate (transverse direction)
z	vertical coordinate, lever arm of internal forces
ΔL	elongation of beam element
ΔN_{cr}	axial force increase resulting from specific fracture energy
ΔV_{R0}	predicted difference of punching shear resistance or portion of punching resistance borne by concrete
ΔV_{R1}	predicted difference of punching shear resistance (strengthened slab)
Δm_{cr}	moment increase resulting from specific fracture energy
Δm_{unl}	moment offset of unloading path
Δm_{rel}	moment offset of reloading path
$\Delta\varepsilon_0$	characteristic tension stiffening effect (TCM)
$\Delta\varepsilon_{cr}$	strain offset
$\Delta\chi_{cr}$	curvature offset
$\Delta\varepsilon_{unl}$	additional tension stiffening for unloading (tension chord)
$\Delta\chi_{unl}$	additional tension stiffening for unloading (slab)
$\Delta\chi_m$	tension stiffening curvature offset (Marti, Burns)
$\Delta\chi_{TS}$	tension stiffening curvature offset (CSCT)
$\Delta\psi_{R0}$	predicted slab rotation increase at failure (non-strengthened slab)
$\Delta\psi_{R1}$	predicted slab rotation increase at failure (strengthened slab)
$\Delta\psi_{UR}$	slab rotation increase after complete UR cycle
$\Delta\psi_{UR,exp}$	experimental slab rotation increase after complete UR cycle
$\Delta\psi_{UR,mod}$	predicted slab rotation increase after complete UR cycle
α_{cyl}	angle of loading cylinder to main axis
β_E	reduction factor of steel Young's modulus
δ	bond slip = relative displacement between steel and concrete
δ_y	slip at which rebar has reached its yield strength
δ_u	slip at which rebar has reached its ultimate strength
ε_c	concrete strain
$\varepsilon_{c,el}$	concrete strain in elastic part of beam
ε_s	steel strain
ε_{sm}	average steel strains over crack element
ε_{sy}	steel strain at yielding stress
κ_V	strength reduction factor for reinforcement inside shear crack
λ	coefficient for crack spacing
ν	Poisson's ratio
ρ	geometrical reinforcement ratio
ρ_m	average reinforcement ratio of both orthogonal directions
$\rho_{s,ef}$	reinforcement ratio relating to effective tension area
σ_c	concrete stress
σ_s	steel stress
σ_{sr}	steel stress at crack edge
σ_{sr0}	steel crack stress
τ_b	bond shear stress between steel and concrete
τ_{b0}	initial bond stress for elastic reinforcement $= 2f_{ct}$
τ_{b1}	initial bond stress after onset of yielding $= f_{ct}$
χ	curvature
χ_{cr}	curvature at cracking
$\tilde{\chi}_{cr}$	curvature at critical crack width w_{cr}
χ_{crs}	curvature at stabilized crack phase (at m_{cr})