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Creep analysis of compact cross-sections cast in consecutive stages – Part 2: Algebraic methods

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ABSTRACT

This paper introduces an application (based on the algebraic methods) that can be used to evaluate the stress and strain time evolution of concrete compact cross-sections cast and/or prestressed in consecutive stages and exposed to long term loading.

The overall cross-section may consist of a concrete, structural steel or fibre reinforced polymer component combined with the original concrete cross section at a distinct stage of the construction period or during the life of the structural element. Moreover, the cross-section can be prestressed both before and after gaining the final shape. Therefore, the application suggested in this manuscript applies to new structures cast in consecutive stages, and to specific problems involved with the rehabilitation or strengthening of concrete structures.

The problem is solved by means of the force method and McHenry's superposition principle.

In a following paper, a series of examples will be utilised to verify the accuracy of the application presented in this document. The results obtained by the method described in this manuscript will be compared to a refined and complex general approach introduced in a previous paper.

1. Introduction

Over the past few decades, construction practitioners have been facing the challenges of high quality demand and high labour cost. In order to overcome these challenges practitioners often develop structures that combine precast elements with cast in place concrete components. The use of this technique gives a monolithic quality to the structure (see for instance [1]).

Similarly, many countries aspire to maintain the traditional identity of built-up areas. As a result, local administrators promote structural retrofitting of existing reinforced concrete structures to enhance their earthquake resistance, to improve their strength to meet new structural demands or new code requirements, and to retrofit damaged structural elements. Moreover, structural retrofitting is adopted to overcome insufficient strength of the materials in new concrete structures resulting from oversight errors and lack of proper quality control. A common technique adopted to improve the bearing capacity of structural elements is to increase the reinforced concrete cross-section. Concrete jacketing of beams and columns is a specific method used to increase the cross-sectional area (see for instance [2–5]).

In all these cases a reliable evaluation of the stress redistribution that occurs in the cross section because of creep and shrinkage of concrete is important to guarantee an accurate forecast of the behaviour of the structure under service loads and at the ultimate load (see [6]).

A general approach utilised to evaluate the stress and strain time evolution of concrete compact cross-sections cast or prestressed in consecutive stages under long term loading was presented in a previous paper [7]. The overall cross-section was made of reinforced concrete, prestressed concrete or steel parts added at distinct stages of the construction process. Moreover, the cross-section could be prestressed several times during construction and after gaining the final shape. This approach led to a system of Volterra integral equations (see for instance [8,9]), whose convolution integral (that is the closed form solution) cannot be determined because of the complexity of the creep function usually adopted to describe concrete behaviour ([10,11]). The system of Volterra integral equations was therefore solved by means of a refined step-by-step time integration method (based on the techniques suggested by classic numerical analysis [12]). The method gives rise to an error whose value can be minimised through a suitable choice of the time discretization procedure. This approach is complicated and cumbersome, hardly implementable in a computer program and too complex for a common engineer. Therefore, this paper illustrates simplified versions of the algebraic

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Notation

x_1 and y_1 principal axes of piece of concrete 1 (the first to be cast)
A_{c1} and J_{c1} area of piece of concrete 1 and its second moment of area with respect to the x_1 axis. The cross-section change because of grouting of tendon 1 (when post-tensioned) is not taken into account
A_{p1} cross-sectional area of tendon 1
x_2 and y_2 principal axes of piece of concrete 2 (the second to be cast)
A_{c2} and J_{c2} area of piece of concrete 2 and its second moment of area with respect to the x_2 axis. The cross-section change because of grouting of tendon 2 is not taken into account
A_{p2} cross-sectional area of tendon 2
E_{c1} and E_{c2} reference elastic moduli (for instance the elastic moduli at the age of 28 days) of concrete 1 and 2 respectively
$E_{c1}(t)$ and $E_{c2}(T)$ elastic moduli of concrete 1 at age t and of concrete 2 at age T respectively
E_{p1} and E_{p2} elastic moduli of the tendons
G_1 centroid of piece of concrete 1
G_2 E-weighted centroid of the final cross section
r ratio between the relaxation loss and the initial prestressing of the prestressing steel
r age of piece of concrete 1
T age of piece of concrete 2
y_{p1} position of tendon 1 on y_1 axis
y_{p2} position of tendon 2 on y_2 axis
y_{c1} position of ΔX_{II} on y_1 axis (therefore y_{c1} is negative in Fig. 2)
y_{c2} position of ΔX_{II} on y_2 axis
ΔX_I^{ld} stress resultant variation in tendon 1, positive when acting according to Fig. 2, caused by the "ld" load
ΔX_{IV}^{ld} stress resultant variation in tendon 2, positive when acting according to Fig. 2, caused by the "ld" load
ΔN_i and ΔM_i^* internal axial force and bending moment variation due to an external long term load. These axial force and bending moment usually follow from a linear elastic structural analysis and therefore act in the centroid of the cross section, i.e. point G_1 or G_2 depending on the current stage of construction. These vectors are positive when acting according to Fig. 2
ΔN_i and ΔM_i internal axial force and bending moment variation due to any external long term load acting at point G_1 (i.e. $\Delta M_i = \Delta M_i^* - \Delta N_i \cdot y_{\Delta N_i}$ when ΔN_i acts at point G_2, $\Delta M_i = \Delta M_i^*$ otherwise. See Fig. 2). These vectors are positive when acting according to Fig. 2
ΔX_{II}^{ld} and ΔX_{III}^{ld} stress resultants in piece of concrete 2 (and in tendon 2, if any) measured on the contact surface between the two pieces of concrete, caused by the "ld" load
δ_{jk}^{sec} term of the flexibility matrix: the axial strain (or the curvature) present in the homogeneous piece "sec" of the cross section, in the point where ΔX_j acts (positive when concordant to ΔX_j), due to $\Delta X_k = 1$
δ_j^{ld} non-compatible strain (or non-compatible curvature) on the contact surface where ΔX_j acts, caused by the "ld" load (positive when acting according to ΔX_j)
$\chi_{c1}(t, t_0^*)$ and $\chi_{c2}(T, T_0^*)$ aging coefficients of concrete 1 and 2 respectively
$\varphi_{c1}(t, t_0^*)$ and $\varphi_{c2}(T, T_0^*)$ creep coefficients of concrete 1 and 2 respectively

methods discussed in [13–15] that allow to overcome the inability to solve the complex numerical integration.

In a following paper the output of the computer program, written according to the more refined solution suggested in the previous paper [7], will be compared with the outcomes of this approach to verify the accuracy of the latter.

2. The approach to problem-solving

The assumptions adopted in the following are:

1. The cross-section is made of two individual homogeneous pieces of concrete (indexes $c1$ and $c2$) or another generic linear viscoelastic material (or an elastic material when setting its creep coefficient to zero) whose constitutive law is a Volterra integral equation approximated by the following algebraic expression (see Fig. 2):

$$\begin{aligned}\varepsilon_{c1}(x_1, y_1, t) &= \frac{\sigma_{c1}(x_1, y_1, t_0^*)}{E_{c1}(t_0^*)}\varphi_{c1}(t, t_0^*)[1 - \chi_{c1}(t, t_0^*)] \\ &\quad + \frac{\sigma_{c1}(x_1, y_1, t)}{E_{c1}(t_0^*)}[1 + \chi_{c1}(t, t_0^*)\cdot\varphi_{c1}(t, t_0^*)] \\ \varepsilon_{c2}(x_2, y_2, T) &= \frac{\sigma_{c2}(x_2, y_2, T_0^*)}{E_{c2}(T_0^*)}\varphi_{c2}(T, T_0^*)[1 - \chi_{c2}(T, T_0^*)] \\ &\quad + \frac{\sigma_{c2}(x_2, y_2, T)}{E_{c2}(T_0^*)}[1 + \chi_{c2}(T, T_0^*)\cdot\varphi_{c2}(T, T_0^*)] \end{aligned} \tag{1}$$

where time t is the age of piece of concrete 1 (the oldest) and time T is the age of piece of concrete 2, related one another by means of the construction history (see Fig. 1). $E_{c1}(t_0^*)$ and $E_{c2}(T_0^*)$ are the elastic moduli measured at the onset of loading, $\varphi_{c1}(t, t_0^*)$ and $\varphi_{c2}(T, T_0^*)$ are the creep coefficients and $\chi_{c1}(t, t_0^*)$ and $\chi_{c2}(T, T_0^*)$ are the aging coefficients.

Both concrete pieces hold a tendon (subscripts $p1$ and $p2$) whose constitutive law is linear elastic (at least under long term service loads):

$$\varepsilon_{p1}(t) = \frac{\sigma_{p1}(t)}{E_{p1}}; \quad \varepsilon_{p2}(t) = \frac{\sigma_{p2}(t)}{E_{p2}} \tag{2}$$

2. No bond slip can occur among the parts which make up the cross-section (external and unbonded internal prestressing and composite steel-concrete beams with flexible connections are therefore not considered).
3. The Bernoulli–Navier hypothesis (an initially plane beam section which is perpendicular to the beam reference axis remains plane and perpendicular to the beam's axis in the deformed configuration) applies to each individual homogeneous part of the cross-section. This assumption is commonly adopted (and accepted) when dealing with compact cross sections in the service stage (as it is the case of the application presented in the following).
4. The internal axial force and bending moment act on a plane of symmetry of the cross section (out-of-plane bending is not taken into account, not to complicate too much the solving system).

The application of the presented approximate solution is therefore restricted to cross-sections cast in two stages. That is, precast prestressed concrete beams (or steel or timber beams) with a cast-in-situ slab (prestressed or not) or jacketed beams and columns (i.e. the cases most frequently found in practical applications).

The stress and strain of the cross-section will be evaluated in the following time intervals (see Fig. 1 that refers to a precast prestressed beam with a cast-in-situ prestressed slab):