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Generalised element load method for first- and second-order element solutions with element load effect

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ABSTRACT

The finite element method in principle adaptively divides the continuous domain with complex geometry into discrete simple subdomain by using an approximate element function, and the continuous element loads are also converted into the nodal load by means of the traditional lumping and consistent load methods, which can standardise a plethora of element loads into a typical numerical procedure, but element load effect is restricted to the nodal solution. It in turn means the accurate continuous element solutions with the element load effects are merely restricted to element nodes discretely, and further limited to either displacement or force field depending on which type of approximate function is derived. On the other hand, the analytical stability functions can give the accurate continuous element solutions due to element loads. Unfortunately, the expressions of stability functions are very diverse and distinct when subjected to different element loads that deter the practical applications. To this end, this paper presents a displacement-based finite element function (generalised element load method) with a plethora of element load effects in the similar fashion that never be achieved by the stability function, as well as it can generate the continuous first- and second-order elastic displacement and force solutions along an element without loss of accuracy considerably as the analytical approach that never be achieved by neither the lumping nor consistent load methods. Hence, the salient and unique features of this paper (generalised element load method) embody its robustness, versatility and accuracy in continuous element solutions when subjected to the great diversity of transverse element loads.

1. Introduction

In regard to the second-order behaviours of a beam-column (element displacement and force solutions), Timoshenko and Gere [1] derived a differential governing equilibrium equation of a member subjected to various element loads as known as the analytical approach. The continuous efforts on the structural stability based on the stability function are drawn for a few decades. For example, frame stability (i.e. Horne and Merchant [2]), element load effects on structural stability (i.e. Chen and Zhou [3]), effective length factors (i.e. Duan and Chen [4,5]), etc. Unfortunately, a specific stability function can only solve the element response under a particular element load. It means the stability function lacks of versatility and generality, especially subjected to various element loads.

On one hand, the variational methods of approximation, including Rayleigh–Ritz and Galerkin method, require the solution over the entire domain approximated by the smooth continuous functions, and thereby they suffer from the well-known drawback that the approximation functions for an arbitrary structure with complex geometry are difficult to construct. For the sake of this, the conventional finite element method (Turner et al. [6], Clough [7], Zienkiewicz [8] and Chan and Kitipornchai [9], Kitipornchai et al. [10]) was developed, of which embodies three salient features and, as a natural result, endows with its superiority over the others; (1) the complex domain in geometry of a structure is discretised into the typical simple piecewise elements; (2) the approximation function of each element are the continuous polynomial function that satisfies its boundary conditions commensurate to its field; (3) the nodal solutions of each element are obtained by satisfying the governing equations, such as force equilibrium equation for structural mechanics, in a weighted integral sense. Therefore, the finite element method exhibits its superiority in versatility for complex geometry, simplicity in approximate function of an element and accuracy at the element nodes.

The conventional finite element method can also refer to the h-version, from which the low polynomial degree of an element

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function is derived. Its accuracy and convergence can be improved by refining the mesh or subdomain of the problem with optimal mesh design. The p-version (Peano [11]; Szabo [12]) in contrast fixes the number and size of finite element, but increases the order of approximate function. Most of the scholars on the topic of p-version, including Babuska et al. [13], focused solely on the numerical convergence based on the error function. Babuska and Suri [14] presented a hybrid h–p version of the finite element method that the convergence is attained by simultaneously refining the mesh and increasing the order of the approximate function of an element. Zienkiewicz et al. [15] and Guo and Babuska [16,17] demonstrated the h–p version embodies not only the higher convergence rate, but also adaptive characteristic for various complex problems. Zienkiewicz et al. [15] illustrated the desired accuracy of nodal values can be attained within one and at most two h–p refinements of plane stress or plane strain elements. To emphasise the engineering applications relatively more than the others aforesaid, Bardell [18] presented the h–p version of Euler–Bernoulli beam element based on the stiffness formulation, in which he demonstrated the accuracy in terms of both deflections and forces within an element under arbitrary distributed loads. However, the accuracy of deflections and forces only confine to the linear elastic behaviour of the beam element.

To attain the accuracy in either displacement or force should respectively resort to the displacement-based element (stiffness approach) or force-based element (flexibility approach), where equilibrium is satisfied only in a weighted integral sense. For the flexibility method, the equilibrium is strictly enforced along the interpolation of internal forces within an element for geometrically linear problems. Kaljevic et al. [19] presented an integrated force method, which simultaneously impose equilibrium equation and compatibility conditions, and produced linear elastic force and deflection solutions. Petrangeli and Ciampi [20] first addressed the major difficulty in developing flexibility approach is the element state determination procedures for nonlinear problem, when they based on the force-based algorithm, in which produced the deflection and force solutions, but only force solutions within an element have been reported and the nonlinear effects accounted for did not specify in detail in their numerical examples. Another flexibility approach developed from Barham et al. [21] enables to capture linear inelastic behaviour at the node, and the refine mesh is required for high level of accuracy.

In contrast with either the stiffness or flexibility approach, both internal forces and displacements are interpolated independently in the mixed formulation, including Hjelmstad and Taciroglu [22] and Alemdar and White [23], upon which the variational principle is relied for both independent dependent variables of force and displacement fields, and the equilibrium and compatibility condition are satisfied in each field. Their second-order inelastic solutions are available at the node merely and the mesh refinement for their approaches is highly recommended.

Some researchers (i.e. Kaljevic et al. [19]; Barham et al. [21]; Alemdar and White [23]) have developed formulations for the nonlinear analysis by using linear or constant interpolation for moments along an element, which corresponds to the solution of the linear equilibrium without element load applied within an element and therefore their accuracy can only be reached at the element node. To take the element load effect into account that is scarce in the open literature, the present approach develops the continuous higher-order element within its domain, but satisfying boundary conditions of both fields in contrast with the salient feature of finite element method (2) and yields the accurate solutions along an element in contrast with the salient feature (3). Eventually, the present higher-order element enables to capture the accurate second-order elastic solutions for a whole domain without element in contrast with the salient feature (1). Therefore, this generalised element load method can be regarded as a great improvement and facilitation of the conventional finite element method as well as the stability function in terms of generality, which is discussed in Section 4.

2. Displacement-based function of higher-order element with element load effect

The deformations comprise the deformations u in the x direction, v in the y direction, w in the z direction and the twist ϕ about the x-axis. The displacement functions of axial deformation u and twist ϕ are assumed linear. Based on the co-rotational coordinate system, the dependent variables of transverse deflections v and w are replaced by nodal rotations as θ_z and θ_y, about z- and y-axis, respectively, such as $\mathbf{u} = \{u, \theta_y, \theta_z, \phi\}^{\mathrm{T}}$. These rotations are the dependent variables in turn which define the transverse deflections in the element stiffness formulation.

External lateral loads acting on an element are able to generate the nonlinear elastic deflections, and thereby the additional deflection component due to transverse element load effect is taken into account of the displacement interpolation function of finite element. This kind of element load can result in the distribution of bending moment and shear force along an element, in which equivalent mid-span moment M_0 and shear force S_0, in Eqs. (4) and (5) respectively, is introduced without loss of generality. Therefore, the higher-order transverse displacement interpolation function of an element not only fulfils the essential boundary condition (compatibility condition) in Eqs. (2) and (3), but also natural boundary condition (force equilibrium equation) in Eqs. (7) and (8) similar to the approach of Chan and Zhou [24]. In this sense, this proposed function can achieve a higher degree of compatibility and improved equilibrium condition at the element level. Further, the elastic material law follows in the higher-order element function.

$$v(x) = \sum_{i}^{p} c_i x^i \tag{1}$$

in which c_i is unknown coefficient solved from boundary conditions given from Eqs. 2–8; p is polynomial of order up to 5 in this sense. In the transverse deflection v in the y direction,

$$v = 0 \quad \text{and} \quad \frac{\partial v}{\partial x} = \theta_{z1} \quad \text{at } \zeta = 0 \tag{2}$$

$$v = 0 \quad \text{and} \quad \frac{\partial v}{\partial x} = \theta_{z2} \quad \text{at } \zeta = 1, \tag{3}$$

while the equilibrium equation of bending and shear force given by

$$EI_z \frac{\partial^2 v}{\partial x^2} = Pv - M_{z1}(1-\zeta) + M_{z2}\zeta + M_0 \tag{4}$$

$$EI_z \frac{\partial^3 v}{\partial x^3} = P\frac{\partial v}{\partial x} + \frac{M_{z1}+M_{z2}}{L} + S_0 \tag{5}$$

$$\text{where} \quad \zeta = \frac{x}{L}. \tag{6}$$

$$EI_z \frac{\partial^2 v}{\partial x^2} = Pv + \frac{M_{z2}-M_{z1}}{2} + M_0 \quad \text{at } \zeta = 1/2 \tag{7}$$

$$EI_z \frac{\partial^3 v}{\partial x^3} = P\frac{\partial v}{\partial x} + \frac{M_{z1}+M_{z2}}{L} + S_0 \quad \text{at } \zeta = 1/2 \tag{8}$$